Volume 8, Nomor 1, Juli 2026, pp. 1-20

RANGE : Jurnal Pendidikan Matematika
e-ISSN :2685-2373

Local Instruction Theory Linear Inequality of One Variable Based on RME to Cultivate Students' Mathematical Communication

Iryana Muhammad[1], Al Jupri[2]*, Kusnandi[3], Siti Fatimah[4]
[1] Department of Mathematics Education, Universitas Malikussaleh, Aceh
[2,3,4] Department of Mathematics Education, Universitas Pendidikan Indonesia, West Java, Indonesia
*Correspondence: aljupri@upi.edu

Received: August, 24th 2025. Accepted: May 26th 2026. Published: July, 23rd 2026

ABSTRACT

Mathematical communication is a fundamental competency that allows students to interpret, represent, and convey mathematical ideas effectively. This study aims to examine the achievement of students' mathematical communication skills through the application of Local Instruction Theory (LIT) to one-variable linear inequality material within the framework of Realistic Mathematics Education (RME). The research uses a design research approach carried out through two teaching experiments, namely TE-1 with 31 students and TE-2 with 26 students at one of the junior high schools in Lhokseumawe. The results showed a significant increase from TE-1 to TE-2 in three indicators of mathematical communication ability, namely: (1) describing graphs into mathematical sentences (58% → 77%); (2) describe or explain mathematical ideas in writing (45% → 96%); and (3) expressing the situation into mathematical language or symbols and developing a solution strategy (39% → 77%). This increase is influenced by the revision of the learning design, which is carried out cyclically and reflectively, so that students better understand the context of the problem, can reflect on the thinking process, and convey solutions mathematically. These findings provide the implication that LIT-based learning design with an RME approach can be a practical reference for teachers and curriculum developers in designing learning activities that systematically and contextually encourage students' mathematical communication skills. This research also affirms the potential of design research in developing local instruction theories that bridge theoretical insights with classroom learning practices.

Keywords: Linear Inequality of One Variable, Local Instruction Theory, Mathematical Communication Skills, Realistic Mathematics Education,

How to Cite: Muhammad, I., Jupri, A. & Fatimah, S. (2026). Local Instruction Theory Linear Inequality of One Variable Based on RME to Cultivate Students' Mathematical Communication. *Range: Jurnal Pendidikan Matematika*, 8(1), 1-20.

Introduction

Mathematical communication skills are one of the key competencies that not only allow students to understand mathematical concepts, but also organize their thoughts, convey ideas logically and clearly, and evaluate problem-solving strategies used by others (Rohid et al., 2019; NCTM, 2000). These skills include the ability to express mathematical ideas through spoken language, writing, and symbolic representation in a precise manner (Mahadewi et al., 2020; Maryati et al., 2022; Muhammad et al., 2023). More than just communicating answers, students are required to reflect on the thought process, discuss solutions with peers, and use mathematical language that meets academic standards (Baroody, 1998; Corebima et al., 2020). In line with that, mathematical communication also includes the skill of representing ideas in the form of graphs, tables, diagrams, and symbols, thus helping students transform abstract ideas into more concrete and easy-to-understand forms (Setiawati et al., 2018; Suhenda & Munandar, 2023). Research shows that good mastery of mathematical communication contributes to improved learning outcomes, critical thinking skills, and the ability to solve problems creatively (Putri et al., 2022; Dwi Indriani & Pasaribu, 2022). Thus, mathematical communication skills can be seen as a multidimensional competency that integrates information interpretation, idea conveyance, and symbolic representation, and becomes an important foundation for students to build a deep conceptual understanding while associating mathematics with real-life contexts (Kamid et al., 2020).

Mathematical communication skills include several important components, namely: (1) the ability to interpret mathematical information, (2) the ability to convey mathematical ideas orally or in writing, and (3) the ability to use mathematical representations such as graphs, diagrams, or symbols. Each of these components contributes to the way students construct and present their understanding of mathematical concepts. The importance of communication in mathematics learning, both in oral and written form (Sampel & Lincoln, 2009;Yang, 2024). Students are encouraged to explain their thought processes, discuss solutions with peers, and use appropriate mathematical language (Baroody, 1998). Thus, mathematical communication skills are needed by students to find and create a tool or way of thinking to communicate mathematical ideas from abstract to concrete, so that they are easier to understand (Setiawati et al., 2018) In learning practice, especially the material of Linear Inequality of One Variable, there are fundamental problems in its presentation in the curriculum and textbooks. Most junior high school packet books place Linear Inequality of One Variable as a subtopic of One-Variable Linear Equations, thus describing it as if students can instantly transfer understanding without a hitch. In fact, at this stage, students are only beginning to get acquainted with the concept and notation of inequality, which are semantically and conceptually different from linear equations.

These conditions show the existence of didactical obstacles, epistemological obstacles, and ontogenetic obstacles. Didactical obstacles arise due to the lack of a structured learning design, epistemological obstacles related to students' limited understanding of the meaning of symbols, graphic representations, and the ability to translate context into mathematical forms, while ontogenetic obstacles are caused by the mismatch between student readiness and task demands. Therefore, these challenges require the development of Local Instruction Theory (LIT) that specifically directs students' learning trajectories, rather than simply applying general approaches such as RME. This gives rise to various misconceptions; For example, in interpreting signs of inequality, drawing wrong solution areas, and difficulties in developing a systematic settlement strategy. All of these barriers weaken students' overall mathematical communication skills.

International research has shown that the Realistic Mathematics Education (RME) approach can improve students' mathematical representation and communication skills (Yuanita & Zakaria, 2016; Muhtarom et al., 2019; Prahmana et al., 2020; Muhammad et al., 2025). Other studies, such as (Muhammad et al., 2025), also report that the development of RME-based worksheets is effective in encouraging the improvement of students' mathematical communication in Indonesia. Some studies published in Range: Journal of Mathematics Education also support the importance of an explicit approach to mathematical communication. For example (M. A. Y. Corebima et al., 2020) found that the Problem-Based Learning (PBL) model improves students' mathematical communication skills in linear program topics. Other research by (E. I. M. Faot et al., 2020) report improved mathematical communication through the Peer Tutoring model.

In the framework of design research, Hypothetical Learning Trajectory (HLT) and Local Instruction Theory (LIT) have a close and sequential relationship. Hypothetical Learning Trajectory (HLT) is not merely a predictive design of learning objectives, activities, and students' thinking, but is grounded in the principles of Realistic Mathematics Education (RME). The characteristics of RME guided reinvention, emergent modeling, and interactivity are embedded in the HLT through the use of contextual problems, the progression from model-of to model-for, and classroom interaction. Thus, the HLT functions as a theoretically informed design that supports the development of a Local Instruction Theory (LIT) Through a series of cycles of design, implementation, and retrospective analysis, the various HLTs that were tested and revised were then accumulated into a more established instruction framework, namely LIT. Thus, methodologically HLT is present first as a contextual hypothesis design, while LIT is the result of generalizations of the HLT cycle that have been empirically validated. However, in practice, LIT can serve as a conceptual reference for teachers to design new HLTs according to the conditions of their respective classrooms. To further demonstrate the difference between HLT and LIT (Gravemeijer & Eerde, 2009;Simon, 1995). Until now, research that specifically develops Local Instruction Theory (LIT) on the material of one-variable linear inequality is still very limited. Research on inequality generally focuses on students' misconceptions and symbolic understandings in mathematics learning (Booth et al., 2020) , while research on LIT in the framework of design research is mostly carried out on other mathematical topics and emphasizes the development of learning trajectories through iterative design cycles (Prediger et al., 2020;Bakker, 2021) . This is a significant gap that this research aims to fill, namely, designing and testing a systematic RME-based LIT to foster students' mathematical communication. In line with that, this study aims to: (1) describe the achievement of students' mathematical communication skills through two stages of teaching experiments; (2) identify improvements in RME-based LIT learning design that have an impact on mathematical communication skills; and (3) demonstrate the practical implications of the RME-based LIT approach as a pedagogical reference for teachers and curriculum developers.

Methods

This research uses a cyclical and iterative design research approach, covering three main stages, namely: (1) Preliminary Design, (2) teaching experiment (TE), and (3) retrospective analysis (Gravemeijer & Cobb, 2006; Bakker, 2018). The design research approach was chosen because it aligns with the cyclical nature of the study's objectives, namely designing a Hypothetical Learning Trajectory (HLT), testing it through teaching experiments, and revising it to produce a valid Local Instruction Theory (LIT). Its iterative and reflective characteristics make this approach appropriate for addressing the research objectives while also generating a contextual local instruction theory.

The use of teaching experiments was considered appropriate since the study focuses on testing the learning design in real classroom settings. Through the teaching experiments, the researcher was able to explore how the predicted HLT worked in practice, how students responded to RME-based activities, and how teacher interventions supported students' mathematical communication. To make the methodological contribution clearer, the mapping between research objectives, methods, and instruments is presented in Table 1.

Table 1. Alignment of Research Objectives, Methods, Instruments, and Data Analyses

Research Objective	Method	Instrument/Data	Data Analysis
Designing HLT toward LIT	Design research (preliminary design & TE)	Draft HLT, initial observations	Qualitative analysis
Implementing RME-based LIT	Teaching experiment (TE-1 & TE-2)	Mathematical communication test, classroom observation	Descriptive quantitative & qualitative analysis
Evaluating supporting and hindering factors	Retrospective analysis	Interviews, field notes	Thematic analysis

This research was conducted in two public junior high schools in Lhokseumawe City. One school became the location of Teaching Experiment 1 (TE-1) with 31 grade VIII students, while the other school became the location of Teaching Experiment 2 (TE-2) with 26 grade VIII students. This separation of locations is intended not to achieve statistical generalization, but to enrich the contextual understanding of the learning design by testing it in different classroom settings. In line with the nature of design research, this approach aims to examine the adaptability and refinement of the learning design rather than to compare or generalize results across contexts. Each cluster consists of three meetings related to the material of One-Variable Linear Inequality, which is detailed in Figure 1 below:

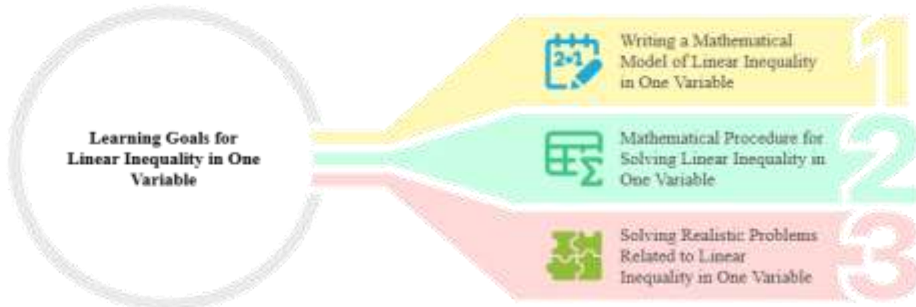


Figure 1. *Linear Inequalities of One Variable learning objectives sequence*

Based on the details of the learning objectives that will be achieved at the three meetings, at the preparation stage, the researcher designed a Hypothetical Learning Trajectory (HLT) for each learning objective, which contains the sequence of learning activities and predictions of student responses (Simon, 1995), Figure 2.

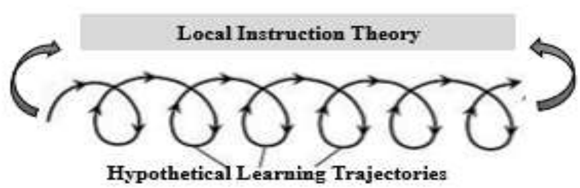


Figure 2. *Development of Hypothetical Learning Trajectory (HLT)*

HLT was implemented in a Teaching Experiment (TE) and analyzed iteratively. HLT transforms into Local Instruction Theory (LIT) when it meets the criteria of stability of learning trajectory, recurrence of student response patterns, and consistency of teacher intervention. If these criteria have not been met, then the LIT produced is still temporary and the HLT will be revised and retested until a valid LIT is obtained.. The research design is summarized in Figure 3 below.

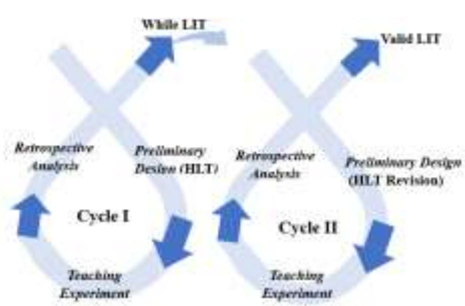


Figure 3. *Research Design Stages*

Based on Figure 3. After the teaching experiment, a retrospective analysis was carried out and revised to produce a better learning design for the next Teaching Experiment (TE-2). Through this cycle, researchers can test, modify, and validate learning designs repeatedly to produce Local Instruction Theory (LIT), which is a local instruction theory that describes students' learning paths, effective activities, and forms of pedagogical intervention that are able to support the development of mathematical communication skills (Gravemeijer & Eerde, 2009; Akker et al., 2007)

The research instruments include a mathematical communication test, observation sheets, and documentation of students' work. Quantitative data were analyzed descriptively (percentage attainment per indicator), while qualitative data were analyzed through reviews of classroom processes, interactions, and reflective notes. The test's content validity was examined by five expert validators using a rubric. However, within design research, the test serves only as a supporting instrument to capture the impact of the learning design, not as the primary tool for developing LIT. In line with process validity, the main focus lies in the iterative refinement of the learning trajectory. Thus, quantitative measures are used to support instrument credibility, not to determine the validity of the LIT.

H_0: Experts' judgments are consistent (no differences).

H_1: Experts' judgments differ.

Decision rule: retain H_0 if Asymp. Sig ≥ 0.05; reject H_0 if Asymp. Sig < 0.05. The computed results of the content-validity test for the mathematical communication assessment are presented in Table 2.

Tabel 2. *Results of the Content Validity Assessment of the Mathematical Communication Test*

Test Statistics	
N	6
Cochran's Q	3.000[a]
df	3
Asymp. Sig.	.392

a. 1 is treated as a success.

In Table 2, the Asymp. Sig. value is 0.392, which is greater than 0.05. Therefore, we fail to reject H_0 (H_0 is accepted), indicating that the experts provided consistent judgments regarding the content validity of the mathematical communication test items.

Empirical validity was established via a pilot with 25 students. To examine item validity and reliability an empirical tryout was conducted with 29 Grade-VIII students at SMP Negeri 5 Lhokseumawe. The results of the item validity analysis are shown in Table 3.

Tabel 3. *Item Validity of the Mathematical Communication Test*

Item	*r* (calculated)	*r* (critical)	Sig.	Decision
KK1	0.788	0.367	0.001	Valid
KK2	0.764	0.367	0.001	Valid
KK3	0.647	0.367	0.001	Valid
KK4	0.611	0.367	0.001	Valid
KK5	0.765	0.367	0.001	Valid
KK6	0.487	0.367	0.007	Valid

Test reliability was estimated using Cronbach's α. Cronbach's alpha was used with the criterion $\alpha > 0.60$. The SPSS 27 output is presented in Table 4 and indicates acceptable internal consistency.

Table 4. *Reliability of the Mathematical Communication Test*

Reliability Statistics	
Cronbach's Alpha	0.768
N of Items	6

According to Table 4, the test results show that the Cronbach's Alpha coefficient for the six items of the mathematical communication test was 0.768. This value indicates that $Cronbach's\ Alpha > 0{,}6$ (Arikunto, 2013; Azwar, 2012; Sugiyono, 2019).

Results and Discussion

Before explaining the results of students' mathematical communication skills, the Hypothetical Learning Trajectory (HLT) was first presented, which was designed and implemented in two teaching experiment cycles. In Teaching Experiment 1 (TE-1), the initial HLT was tested on students to see the suitability of the learning design with their responses and thought processes. From the implementation of TE-1, Local Instruction Theory 1 (LIT 1) was produced, which is a local instruction theory that summarizes learning paths, activities, and effective pedagogical strategies based on empirical experience in the classroom. Figure 4 below presents the first learning objective, which is to write the Linear Inequalities in One Variable mathematical model:

Volume 8, Nomor 1, Juli 2026, pp. 1-20

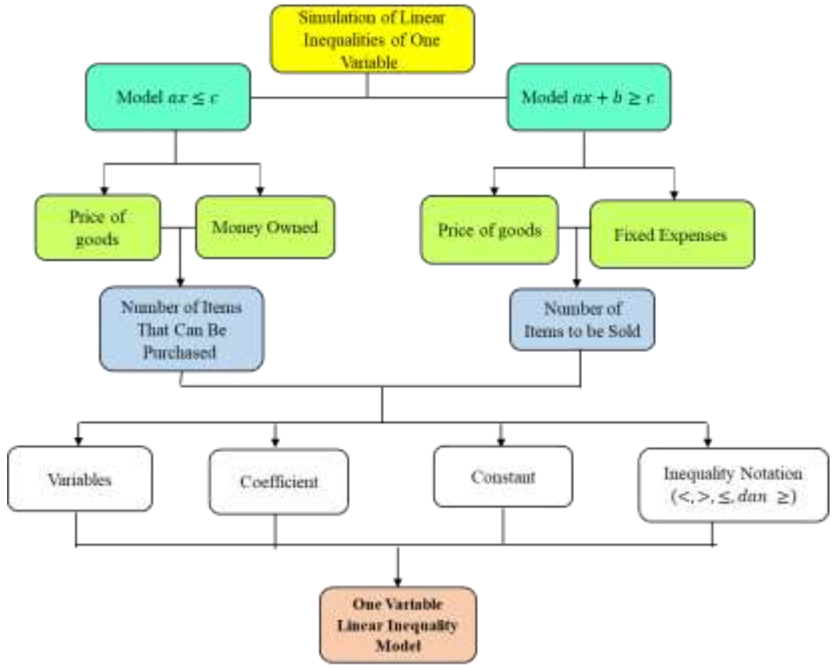


Figure 4. *LIT Flow Model Linear Inequalities of One Variable Cycle I*

Based on Figure 4. The flow in the diagram represents learning activities in achieving the goal of creating a single-variable linear inequality model. Students are invited to simulate contextual situations, such as the relationship between the price of goods and the money they have or the number of goods that must be sold with fixed expenses, and then reduce them to a form of $ax \geq b$ or $ax + b \leq c$ inequality. Through this process, students identify the main components, namely variables, coefficients, constants, and inequality notations (<, ≤, > or ≥), so that in the end they can compile a systematic Linear Inequalities in One Variable model. Furthermore, Figure 5 shows the second learning activity: finding the procedure for completing Linear Inequalities in One Variable mathematically. Figure 5 indicates LIT for the second purpose.

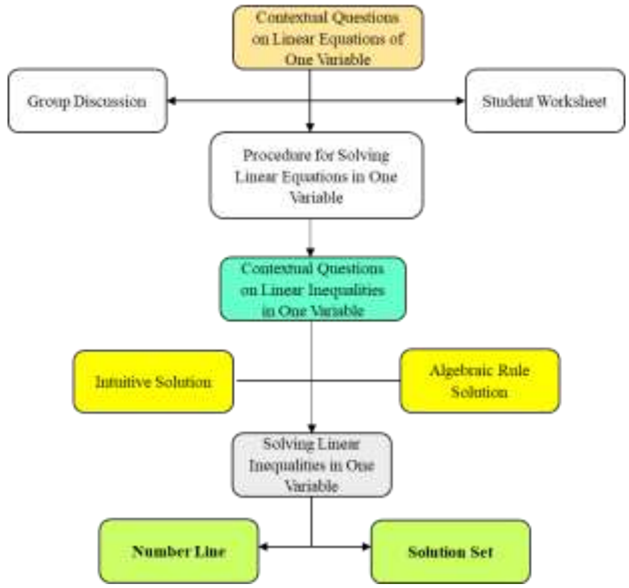


Figure 5. *LIT Flow Linear Inequalities of One Variable Completion Procedure Cycle I*

The diagram shows the learning activity of solving a Single Variable Linear Inequality procedurally. Students are invited to contextual questions of linear equations through group discussions and the Student Worksheet, then to Linear Inequalities in One Variable contextual problems, which are solved by intuition and algebraic rules. The result is expressed as a complete mathematical procedure, indicated by the number lines and the set of solutions. The following is shown in Figure 6.

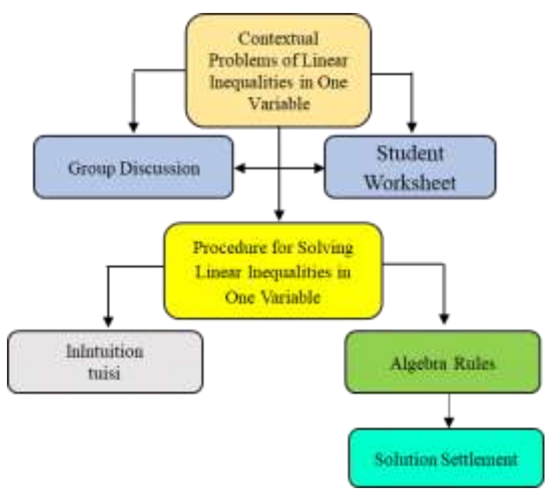


Figure 6. *Cycle I Linear Inequalities of One Variable Completion LIT Flow*

Figure 6 shows the flow of the learning activity of the realistic one-variable linear inequality problem-solving learning activity. Students are given contextual problems that are close to real life, and then discuss them through the Student Worksheet. From there, students are directed to the Linear

Inequalities in One Variable completion procedure, both by intuition and using algebraic rules, until finally finding a solution that suits the context.

In addition to the next cycle 1 LIT, the results of TE-1 also reveal weaknesses and obstacles, especially related to student misconceptions and limitations in conveying mathematical ideas. Therefore, the HLT was revised based on retrospective findings to address these weaknesses. This revision was then implemented in Teaching Experiment 2 (TE-2), resulting in LIT 2, which is a local instruction theory that has been enriched and strengthened through a cycle of improvement. Thus, LIT 2 can be seen as the final form of the design research process, which is more stable and ready to be used as a reference for learning one-variable linear inequalities in an effort to grow students' mathematical communication skills. The results of TE-1 were then analyzed retrospectively to identify weaknesses, obstacles, and misconceptions experienced by students. This analysis was used as the basis for the revision of the HLT, resulting in a more mature and contextual learning design in Teaching Experiment 2 (TE-2). Thus, HLT not only plays a role as an initial design, but also as a dynamic instrument that continues to be developed cyclically to give birth to a more stable Local Instruction Theory (LIT) and can be used as a reference for the practice of learning one-variable linear inequalities. The LIT in the second cycle for each learning objective is shown in Figures 7, 8, and 9, respectively.

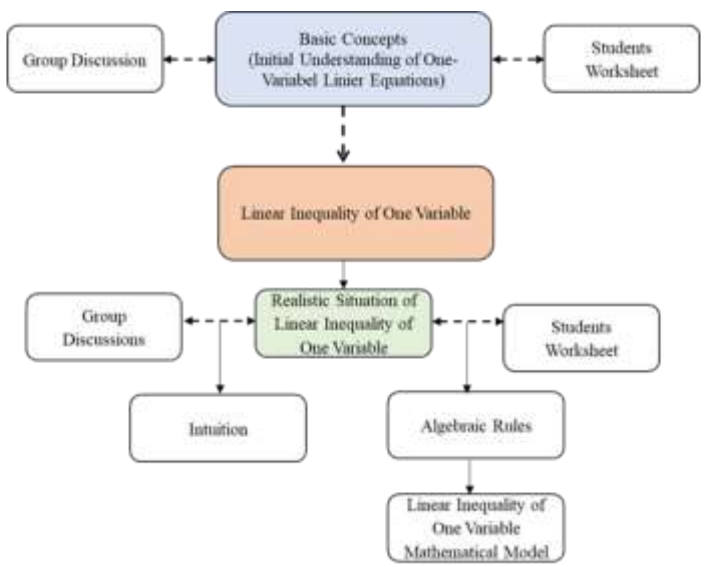
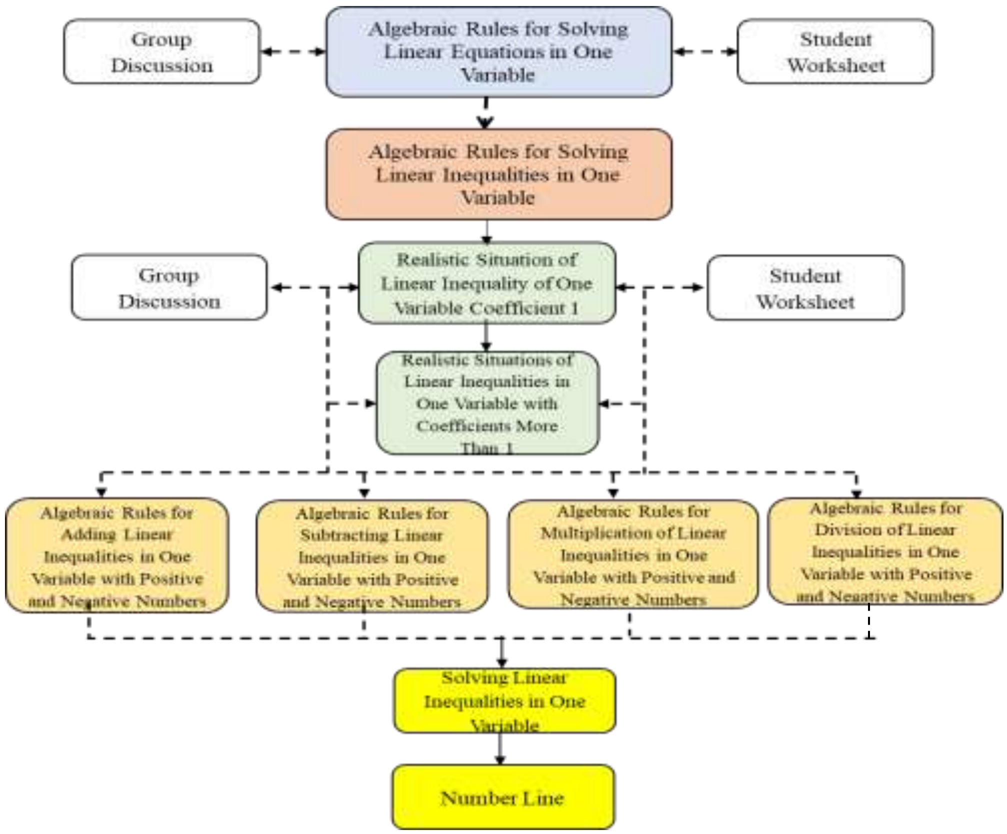


Figure 7. *LIT Flow Model Linear Inequalities of One Variable Cycle II*



Picture. 8. *LIT Flow Linear Inequalities of One Variable Completion Procedure Cycle II*

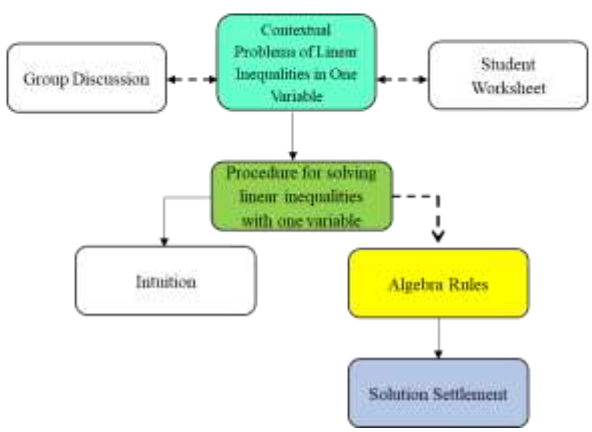


Figura 9 *Cycle II Linear Inequalities of One Variable Completion LIT Flow*

LIT Linear Inequalities in One Variable Design with a Holistic RME Approach

The improvement of LIT formed during cycles I and II reflects that learning process activities have increased from the previous 89.78% to 94%. Based on this percentage, it is a manifestation of the results of the analysis of the implementation of learning in cycle II, which shows that many learning obstacles found in cycle I have been successfully overcome through improvements in the learning design carried out. These improvements not only have an impact on improving students' conceptual understanding of Linear Inequalities in One Variable material but also contribute positively to the development of students' mathematical communication skills. The anticipation process that has been carried out by teachers to the maximum at each learning meeting is described in the following:

a. In the first meeting, the focus of learning was to write a mathematical model in the form of Linear Inequalities in One Variablebased on contextual situations from daily life. In the first cycle, obstacles were found in the form of inaccuracies in arranging mathematical forms, especially in determining the exact position of variables and coefficients. For example, some students still put the value of a variable as a coefficient, as in the answer $9x + 350{,}000 \geq 700{,}000$, even though the number 9 is a variable value, not a multiplier factor. This error shows that the students' understanding of the relationship between contextual information into a formal mathematical form is incomplete. However, in cycle II, these obstacles are no longer visible. Students have been able to analyze the situation more carefully and develop a mathematical model that is more representative of the context of the problem. They demonstrate a deeper understanding of the role of coefficients and variables in the Linear Inequalities in One Variable model. This development indicates an improvement in students' mathematical communication skills, particularly in their ability to construct and interpret symbolic representations from verbal problems. This finding aligns with the concept of mathematical communication, which involves expressing, interpreting, and connecting mathematical ideas (Cooke & Buchholz, 2005; NCTM, 2000b), and is supported by recent studies emphasizing its role in students' conceptual understanding (Booth et al., 2020). In addition, there was an increase in students' self-determination, shown through their efforts to re-verify the mathematical sentences that had been composed.

b. At the second meeting, the material focused on the completion of Linear Inequalities in One Variable and the interpretation of results in number lines. In cycle I, students have difficulty in carrying out algebraic procedures, especially when facing problems with long stages, such as in the form of $3x + 3 > x + 9$. Students sometimes get stuck in the middle of the process, even though they have done the initial steps correctly. In addition, students are also not completely correct in interpreting the results into a representation of number lines. After making design adjustments in cycle II, including the use of illustrative media such as scale analogies and providing visual steps for completion, students showed significant progress. They can solve problems completely and find the final solution independently. The representation of number lines can also be arranged accurately. This change signifies an improvement in students' mathematical procedural and communication skills, and shows the development of their self-determination to solve problems to the final stage.

c. In the third meeting, learning focused on solving Linear Inequalities in One Variable problems from the context of a realistic situation. In the first cycle, students have been quite successful in solving problems that are close to their daily lives, but still have difficulties with problems that require a more in-depth understanding. This difficulty is caused by the need to analyze more complex information from the problem situation. Through learning in cycle II, these barriers were successfully minimized by providing more intensive scaffolding through structured group discussions. Students are directed to understand the structure of information and identify important elements in the question collaboratively. As a result, they can solve all types of problems given with a higher level of accuracy. This shows that through adaptive learning support, both students' mathematical communication skills and self-determination are significantly strengthened.

Overall, the implementation of learning design in cycle II successfully overcame various learning obstacles found in cycle I. These improvements have a positive impact in directing students to think more reflectively, communicatively, and responsibly about the learning process and outcomes achieved. Therefore, based on the three learning objectives of Linear Inequalities in One Variable and the revision of the Linear Inequalities in One Variable LIT resulting from cycles I and II, the overall LIT Linear Inequalities in One Variable is obtained as follows, Figure 10:

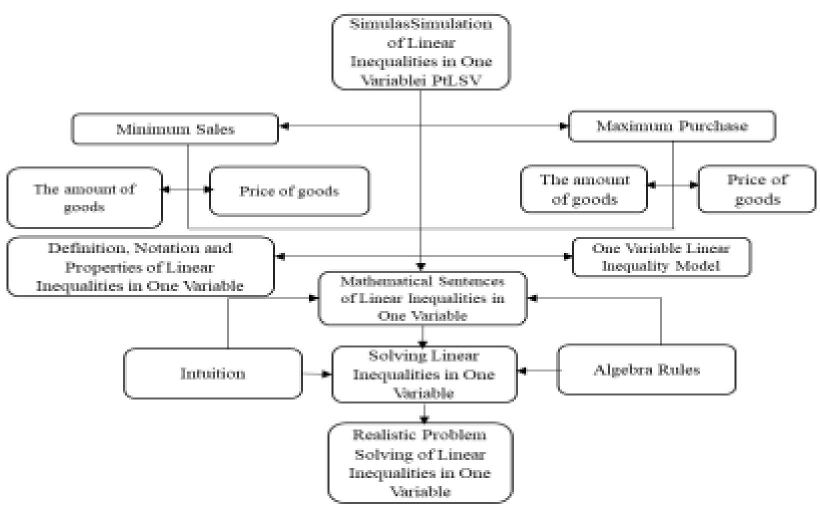


Figure 10. *Linear Inequalities of One Variable LIT Design*

Figure 10, illustrates the trajectory Activities in LIT are arranged as a learning trajectory that gradually develops students' mathematical communication skills. Contextual problems in the early stages encourage students to express ideas informally, which are then refined through interaction and directed discussion. This trajectory facilitates the transition from model-of to model-for, so students move from context to formal mathematical representations. Through this process, students learn to express relationships, use symbols appropriately, and associate various representations, so that mathematical communication develops systematically.

Achievement of Mathematics Communication Skills

The mathematical communication ability test in this study consists of six questions that are systematically arranged based on the indicators of mathematical communication ability. These indicators include: (1) describing graphs into mathematical sentences, (2) drawing or explaining ideas and situations in writing, and (3) expressing situations into mathematical language or symbols and strategizing mathematical problem solving logically. Each question has a maximum score weight of 4, so the maximum total score for the entire test is 16 points. .

The analysis of the results of this test is not focused on inferential statistical testing, but is aimed at finding out the extent of the achievement of each indicator of students' mathematical communication ability in a quantitative descriptive manner, namely through the calculation of the percentage of achievement. Test result data were obtained from 31 students in the implementation of TE-1 and 26 students in TE-2. The percentage of achievement in each indicator will be displayed and analyzed through Table 5, to provide an overview of the development of students' mathematical communication skills during the learning process of the Local Instruction Theory (LIT) design with the Realistic Mathematics Education (RME) approach applied in these two research cycles.

Table 5. *Percentage of Achievement of Mathematical Communication Ability Score Students to Class Average Scores*

Communication Ability Indicators Mathematical	Achievement Percentage	
	TE-1	TE-2
Describe graphs in math sentences	20 of 31 students 65%	21 of 26 students 81 %
Draw or explain ideas and situations in writing	16 of 31 students 52%	24 of 26 students 92%
Declare the situation in language or mathematical symbols and develop a strategy for solving mathematical problems	17 of 31 students 55%	21 of 26 students 81%

Volume 8, Nomor 1, Juli 2026, pp. 1-20

Based on the results of data analysis presented in Table 4.5, it was seen that there was a significant increase in students' mathematical communication skills after the application of the Local Instruction Theory (LIT) design to the Linear Inequalities in One Variable material. First, in the indicator of translating graphs into mathematical sentences, achievement increased from 65% in TE-1 to 81% in TE-2. Beyond this quantitative improvement, qualitative analysis revealed a shift in students' responses: in TE-1, students frequently produced fragmented or incorrect symbolic expressions, whereas in TE-2, their responses became more structured, with more precise symbolization and a clearer articulation of the relationships represented in the graph. Second, indicators of drawing or explaining ideas and situations in writing showed the most marked increase, from 52% to 92%, indicating that the designed learning activities have successfully encouraged students to express their mathematical ideas in writing more clearly and in a structured way. Third, in the indicators of expressing the situation into mathematical language or symbols and developing problem-solving strategies, achievement increased from 55% to 81%, indicating that students were increasingly able to translate contextual problems into symbolic representations and design appropriate problem-solving strategies. The achievement of mathematical communication skills for each cycle is shown visually in Figure 11.

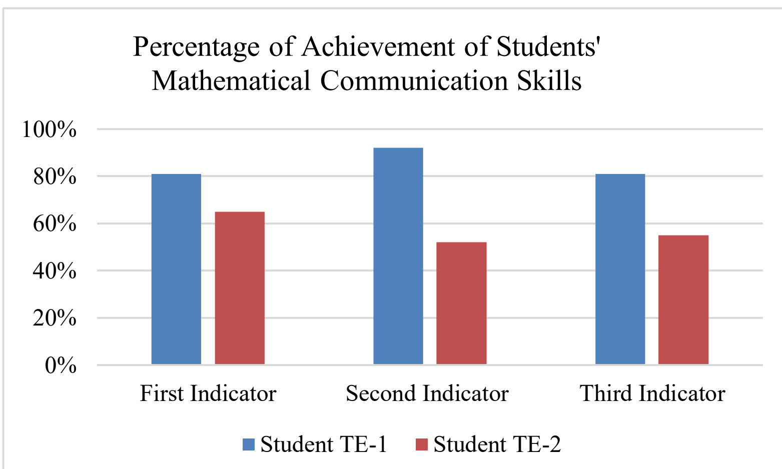


Figure 11*. Percentage of Achievement of Mathematical Communication Ability Score Against Class Average Score*

The results of the students' mathematical communication skills test in the second experimental stage (TE-2) showed a high average of achievement, which reflects the successful implementation of the LIT design. The three main indicators analyzed experienced a significant increase: the indicator of turning a graph into a mathematical sentence increased from 65% to 81%, the indicator of drawing or explaining ideas/situations in writing increased sharply from 52% to 92%, and the indicator of expressing the situation into mathematical language/symbols and formulating a solution strategy increased from 55% to 81%. Thus, all indicators show an increase from TE-1 to TE-2.

This improvement is inseparable from the improvement of the LIT design in the previous stage. The revisions made after TE-1 allow students to better understand the context of the problem, reflect on the thought process, and present solutions mathematically. The obstacles that arise in TE-1 can be minimized so that students are more confident in communicating mathematical ideas in TE-2. This shows that cyclical and reflective LIT design is able to develop students' mathematical communication in a sustainable manner. These findings are in line with previous research. (Fauzan, 2021;Samsinar et al., 2023) affirm that the integration of real context and group discussions is effective in encouraging students to communicate mathematical ideas. (Usolin et al., 2023) show that students' mathematical communication varies based on cognitive style. Although the topics are different from PtLSV, the same dynamics are visible: learning designs that take into account the characteristics of students can strengthen communication effectiveness. Further, (Andika et al., 2025) emphasize the importance of realistic contexts to foster student reflection. This is in line with the implementation of LIT, which emphasizes the transition of model-of to model-for and ends in formal knowledge.

From a theoretical perspective, these results support the principles of RME, especially guided reinvention and emergent modeling. (Gravemeijer & Eerde, 2009b) emphasized that the success of the transition from a situational context to a formal form is largely determined by bridging activities. In this study, the use of the context of shopping and scales serves as a connecting activity that helps students move from informal models to formal representations of One-Variable Linear Inequality.

Thus, this study not only confirms previous studies but also expands on the contribution by producing a Local Instruction Theory (LIT) specific to PtLSV. This LIT describes in detail how the transition from contextual activities to symbolic representations can be designed to develop students' mathematical communication. This contribution reinforces the findings of (Vos, 2018 ;Schoevers et al., 2019;Faot et al., 2020;Chan et al., 2021) who emphasize the importance of local instruction theory as a bridge between mathematics education theory and classroom practice..

Conclusion

This research produces Local Instruction Theory (LIT) based on Realistic Mathematics Education (RME) on the material of one-variable linear inequality which is characterized by a structured learning trajectory and supports the gradual development of students' mathematical communication skills. The developed LIT integrates the main characteristics of RME, namely the use of contextual problems, the development from informal to formal representations, as well as classroom interactions that encourage students to express mathematical ideas. The transformation of Hypothetical Learning Trajectory (HLT) into LIT is shown through the stability of the learning trajectory between implementations, the recurrence of student response patterns, and the consistency of teacher interventions. In this framework, students showed an increase in the ability to express mathematical ideas in writing, which indicates the potential of LIT developed in supporting mathematical communication skills even significantly on students' ability to draw or explain ideas in writing, from 52% to 92%.

These findings show that RME-based LIT can be a theory-based reference and empirically supported in designing mathematics learning that is contextual, interactive, and oriented to students' communication processes (Çiloğlu et al., 2025; Sari et al., 2025; Siswantari et al., 2025). Further research is suggested to test the adaptability of this LIT to other mathematics topics and different levels of education.

Acknowledgement

The authors would like to thank the Pusat Pembiayaan dan Asesmen Pendidikan Tinggi (PPAPT) and Lembaga Pengelola Dana Pendidikan (LPDP), which funded this study.

References

Akker, J. van den, Bannan, B., Kelly, A. E., Nieveen, N., & Plomp, T. (2007). An Introduction to Educational Design Research. *An Introduction to Educational Design Research*.

Andika, R., Rani, M. M., Syam, S. S., & Firdaus, A. (2025). Analysis of Students' Mathematics Communication Ability Through the Realistic Mathematics Education Refiona. *Journal of Practice Learning and Educational Development*, *5*(1), 247–257. https://doi.org/10.58737/jpled.v5i1.402

Arikunto, S. (2013). *Prosedur Penelitian: Suatu Pendekatan Praktik* (Edisi revi). Jakarta: Rineka Cipta.

Azwar, S. (2012). *Reliabilitas dan Validitas*. Pustaka Pelajar.

Bakker, A. (2018). Design Research in Education: A Practical Guide for Early Career Researchers. In *Routledge* (Issue 68). Taylor & Francis. https://doi.org/10.1049/ic:19990398

Baroody, A. J. (1998). *Fostering Children ' s Mathematical Power : An Investigative Approach to K-8 Mathelllatics Instruction*. https://doi.org/10.4324/9781410602084

Booth, J. L., Newton, K. J., & Twiss-Garrity, L. K. (2020). The impact of error-focused instruction on student learning in mathematics. *Journal of Educational Psychology*. https://doi.org/10.1037/edu0000372

Çiloğlu, R., Korkmaz, H., & Mut, A. İ. (2025). Evaluation of 6th Grade Students ' Mathematical Communication Levels within the Framework of Realistic Mathematics Education Approach. *Bartın University Journal of Faculty of Education*, *14*(3). https://doi.org/10.14686/buefad.1687206

Cooke, B. D., & Buchholz, D. (2005). Mathematical Communication in the Classroom: A Teacher Makes a Difference. *Early Childhood Education Journal*, *32*(6), 365–369. https://doi.org/10.1007/s10643-005-0007-5

Corebima, M. A. Y., Garak, S. S., & Samo, D. D. (2020). Pengaruh Problem Based Learning Terhadap

Kemampuan Pemecahan Matematis Siswa Kelas XI SMA. *Jurnal Pendidikan Matematika*, *2*(1), 56–65.

Corebima, M., Garak, S., & Samo, D. (2020). PENGARUH MODEL PROBLEM BASED LEARNING TERHADAP KEMAMPUAN KOMUNIKASI MATEMATIS SISWA PADA MATERI PROGRAM LINEAR. *RANGE: Jurnal Pendidikan Matematika*, *2*, 56–65. https://doi.org/10.32938/jpm.v2i1.569

Faot, E. I. M., Leton, S. I., & Dosinaeng, W. B. N. (2020). Pengaruh Model Pembelajaran Peer Tutoring Untuk Meningkatkan Kemampuan Komunikasi Matematis Siswa. *RANGE: Jurnal Pendidikan Matematika*, *1*(2), 157–163. https://doi.org/10.32938/jpm.v1i2.412

Faot, F., Yerizon, Y., & Zulkardi. (2020). Improving Students' Mathematical Communication Skills Tthrough RME-based L earning. *Journal of Physics: Conference Series*, *1663*(1), 12059. https://doi.org/10.1088/1742-6596/1663/1/012059

Fauzan, A. (2021). The Development of Student Worksheet Based on Realistic Mathematics Education in Ordinary Differential Equations of Order-1. *Journal of Physics: Conference Series*. https://doi.org/10.1088/1742-6596/1742/1/012018

Gravemeijer, K., & Cobb, P. (2006). Design Research From a Learning Design Perspective. *Educational Design Research*, *May*, 17–51. https://doi.org/10.4324/9780203088364-12

Gravemeijer, K., & Eerde, H. A. A. (2009a). Design Research as a Means for Building a Knowledge Base for Teachers and Teaching in Mathematics Education. *Elementary School Journal*, *109*. https://doi.org/10.1086/596999

Gravemeijer, K., & Eerde, D. van. (2009b). Design Research as a Means for Building a Knowledge Base for Teachers and Teaching in Mathematics Education. *The Elementary School Journal*, *109*(5), 510–524. https://doi.org/https://doi.org/10.1086/596999

Indriani, W. D., & Pasaribu, L. H. (2022). *Peningkatan Kemampuan Komunikasi Matematis Siswa Menggunakan Model Pembelajaran Hybrid Learning*. *06*(01), 291–299.

Kamid, Rusdi, M., Fitaloka, O., Basuki, F. R., & Anwar, K. (2020). Mathematical Communication Skills Based on Cognitive Styles and Gender. *International Journal of Evaluation and Research in Education*, *9*(4), 847–856. https://doi.org/10.11591/ijere.v9i4.20497

Mahadewi, N. K. N., Ardana, I. M., & Mertasari, N. M. S. (2020). Kemampuan Komunikasi Matematis Melalui Model Reciprocal Teaching Berbantuan Media Interaktif. *JNPM (Jurnal Nasional Pendidikan Matematika)*, *4*(2), 338. https://doi.org/10.33603/jnpm.v4i2.3606

Maryati, I., Suzana, Y., Harefa, D., & Maulana, I. T. (2022). Analisis Kemampuan Komunikasi Matematis dalam Materi Aljabar Linier. *PRISMA*, *11*(1), 210. https://doi.org/10.35194/jp.v11i1.2044

Muhammad, I., Herizal, H., Marhami, M., Rizkiana, S. T., & Lianti, V. (2023). Pengembangan Buku Matematika SMP Kelas VIII Berbasis Etnomatematika Terintegrasi Kemampuan Komunikasi Matematis. *AKSIOMA: Jurnal Program Studi Pendidikan Matematika*, *12*(1), 882. https://doi.org/10.24127/ajpm.v12i1.6522

Muhammad, I., Juandi, D., & Jupri, A. (2025). Implementation of Ethnomathematics Realistic Mathematic Education (Ethno-RME) in Mathematics Learning: Systematic Literature Review (SLR). *AIP Conference Proceedings*, *March 2025*. https://doi.org/https://doi.org/10.1063/5.0262106

NCTM. (2000a). Principles and Standards for School Mathematics. *In School Science and Mathematics* , *47*(8).

NCTM. (2000b). *Principles and Standards for School Mathematics*. The National Council of Teachers of Mathematics, Inc. www.nctm.org

Prahmana, R. C. I., Sagita, L., Hidayat, W., & Utami, N. W. (2020). *Two Decades of Realistic Mathematics Education Research in Indonesia: A Survey*.

Prediger, S., Confrey, J., & Gravemeijer, K. (2020). Design research with a focus on learning processes: Combining theory building and classroom intervention. *ZDM Mathematics Education*, *52*(1), 1–14. https://doi.org/10.1007/s11858-019-01085-0

Putri, N. S., Juandi, D., & Jupri, A. (2022). The Implementation Effect of Realistic Mathematics Education and Contextual Teaching and Learning Approaches on the Students ' Mathematical Communication Ability : A Meta-analysis. *Al-Jabar: Jurnal Pendidikan Matematika*, *13*(2), 383–400. https://doi.org/http://dx.doi.org/10.24042/ajpm.v13i2.13562

Rohid, N., Suryaman, S., & Rusmawati, R. D. (2019). Students' Mathematical Communication Skills (MCS) in Solving Mathematics Problems : A Case in Indonesian Context. *Anatolian Journal of Education*, *4*(2), 19–30. https://doi.org/https://doi.org/10.29333/aje.2019.423a

Sampel, L., & Lincoln, N. (2009). Oral and Written Communication in Classroom Mathematics. *DigitalCommons@University of Nebraska - Lincoln Action*.

Samsinar, S., Mahmudah, K. R., & Prasetyo, P. W. (2023). Development of Realistic Mathematics Education (RME)-Based Student Worksheet to Improve Students' Mathematical Communication Skills. *International Journal on Emerging Mathematics Education*, *7*(2), 119–132. https://doi.org/10.12928/ijeme.v7i2.28729

Sari, R., Maharani, I., & Harahap, Y. N. (2025). Pengaruh Model Pembelajaran Realistic Mathematic Education untuk Meningkatkan Kemampuan Komunikasi Matematis Siswa Berbantuan Wordwall. *Pendas: Jurnal Ilmiah Pendidikan Dasar*, *10*(1), 223–234.

Schoevers, E. M., Leseman, P. P. M., & Kroesbergen, E. H. (2019). Improving Low-achieving Adolescents' Mathematical Word Problem Solving: A Design Research Intervention Study. *Mathematical Thinking and Learning*, *21*(2), 85–107. https://doi.org/10.1080/10986065.2019.1570833

Setiawati, R., Netriwati, N., & Nasution, S. P. (2018). Desain Model Pembelajaran Gerlach dan Ely yang Berciri Nilai-nilai Ke-Islaman untuk meningkatkan Kemampuan Komunikasi Matematis. *AKSIOMA: Jurnal Program Studi Pendidikan Matematika*, *7*(3), 371–379. https://doi.org/10.24127/ajpm.v7i3.1593

Simon, M. A. (1995). Reconstructing Mathematics Pedagogy from a Constructivist Perspective. *Journal for Research in Mathematics Education*, *26*(2), 114–145. https://doi.org/10.5951/jresematheduc.26.2.0114

Siswantari, Sabon, S. S., Listiawati, N., Wirda, Y., Zulkardi, & Riyanto, B. (2025). Bridging mathematics and communication: Implementing realistic mathematics education principles for skill development. *Journal on Mathematics Education*, *16*(2), 729–752. https://doi.org/10.22342/jme.v16i2.pp729-752

Sugiyono. (2019). *Metode Penelitian Pendidikan: Pendekatan Kuantitatif, Kualitatif, dan R&D* (Edisi ke-3). Alfabeta.

Suhenda, L., & Munandar, D. (2023). Kemampuan Komunikasi Matematis Siswa dalam Pembelajaran Matematika. *Jurnal Educatio FKIP UNMA*, *9*, 1100–1107. https://doi.org/10.31949/educatio.v9i2.5049

Usolin, K., Son, A. L., Maifa, T. S., & García-García, J. (2023). Profile of Mathematics Communication Ability of Seventh-Grade Students in Solving Set Problems Based on Cognitive Style. *RANGE: Jurnal Pendidikan Matematika*, *4*(2), 189–201. https://doi.org/10.32938/jpm.v4i2.3593

Vos, P. (2018). The Role of Context in RME. *International Journal of Science and Mathematics Education*, *16*(1), 145–164. https://doi.org/10.1007/s10763-016-9770-0

Yang, J. (2024). *The Characteristics of Mathematical Communication in Secondary School Students' Collaborative Problem Solving*. Springer. https://link.springer.com/10.1007/978-981-99-7386-6