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Errors and Their Causes in Solving One-Variable Linear Equations Among Mexican High School Students

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Abstract
The aim of this study was to identify the errors made by a group of Mexican High School students when solving linear equations, as well as their possible causes. Data collection was conducted through a task-based interview administered to 30 students from a public school in southern Mexico. The results revealed that students made arithmetic errors when solving linear equations, including difficulties in performing operations with integers, handling fractions, applying the distributive property, and transposing terms incorrectly. By analyzing students' reasoning, it was inferred that these errors stem mainly from poor assimilation of arithmetic concepts, affective and emotional factors, and the presence of cognitive obstacles. Additionally, both procedural and conceptual algebraic errors were identified. According to the conceptual framework, these errors originate from limited manipulation of algebraic language, incorrect application of procedural rules, and didactic and cognitive obstacles. These findings encourage reflection on future research aimed at improving the learning of linear equations at the high school level.

Resumen
El objetivo de este estudio fue identificar los errores evidenciados por un grupo de estudiantes mexicanos de nivel medio superior al resolver ecuaciones lineales, así como sus posibles causas. La recolección de datos se realizó mediante una entrevista basada en tareas aplicada a 30 estudiantes de una escuela pública del sur de México. Los resultados revelaron que los estudiantes cometieron errores aritméticos al resolver ecuaciones lineales, incluyendo dificultades para realizar operaciones con números enteros, manejar fracciones, aplicar la propiedad distributiva y transponer términos incorrectamente. Al analizar el razonamiento de los estudiantes, se infirió que estos errores se derivan principalmente de una mala asimilación de conceptos aritméticos, factores afectivos y emocionales, y la presencia de obstáculos cognitivos. Adicionalmente, se identificaron errores algebraicos tanto procedimentales como conceptuales. De acuerdo con el marco conceptual, estos errores se originan en la manipulación limitada del lenguaje algebraico, la aplicación incorrecta de reglas procedimentales y obstáculos didácticos y cognitivos. Estos hallazgos invitan a la reflexión sobre futuras investigaciones dirigidas a mejorar el aprendizaje de ecuaciones lineales en el nivel medio superior.

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Introduction

For decades, research in mathematics education has focused on understanding the difficulties students face at different levels when learning mathematical concepts. In particular, numerous findings have significantly contributed to improving the teaching and learning of algebra (e.g., Maizora & Juandi, 2022; Nurjannah & Jupri, 2024; Pérez et al., 2019; Socas, 2011).

In this context, research in mathematics education has identified learning difficulties as a key factor that not only hinders students' academic development but also contributes to the formation of negative attitudes toward mathematics (Kaput, 2000). Specifically, it has been recognized that these difficulties are closely related to the occurrence of errors in solving mathematical exercises and problems (Utami & Kusumah, 2024). For this reason, identifying, analyzing, and preventing these errors have been central areas of interest in mathematics education research (Socas, 2011). Various studies have sought to understand the origin of these errors and minimize their impact on learning, with the goal of optimizing teaching processes and strengthening students' mathematical understanding (e.g., Adu et al., 2015; Mengistie, 2020; Pérez et al., 2019; Yansa et al., 2021).

In the Mexican mathematics curriculum, the concept of a linear equation holds a central place, especially at the pre-university level (Ministry of Public Education [SEP, for its acronym in Spanish], 2023; Pérez et al., 2019), as it integrates various algebraic foundations that facilitate the transition from arithmetic to algebra (Molina, 2009; Pirie & Martin, 1997). Furthermore, studying linear equations plays a key role in mathematical knowledge construction, as it serves as the foundation for understanding multiple concepts throughout secondary and high school education (SEP, 2023).

Research on algebra learning has addressed fundamental aspects such as symbolization, generalization, and the development of algebraic reasoning (Molina, 2009). However, difficulties related to solving linear equations remain a recurring issue among students, frequently appearing in mathematics classes, especially at the secondary level (Pérez et al., 2019). Therefore, we start from the hypothesis that these errors not only persist in secondary school but also appear in other educational levels, such as High School Level (NMS, by its acronym in Spanish). However, there is still insufficient empirical evidence to confirm this. For this reason, the present study aligns with this line of research, with the premise that identifying the errors made by NMS students when solving linear equations will help develop more effective teaching strategies to mitigate these persistent difficulties in algebra instruction.

Interpreting the causes of students' mathematical errors is a complex task. In fact, there is no systematic theoretical framework that fully explains their origin (Rico, 1995). So far, most descriptive studies have been based on empirical observation; however, it is essential to complement this approach with a well-founded theoretical perspective that helps clarify the roots of these errors (Abrate et al., 2006). In this context, the present research is relevant as it contributes to a better understanding of the difficulties surrounding linear equations and aligns with the work of Socas (1997) and Pérez et al. (2019).

Additionally, various studies on linear equations have proposed classifications of student errors based on different criteria. Hall (2002), for example, categorizes them according to the level of difficulty of the given equation. In contrast, Ruano et al. (2008) and Egodawatte (2011) focus on errors stemming from conceptual aspects, while Abrate et al. (2006) not only classify these errors but also analyze their possible causes. The complexity of diagnosing the origin of an error has led various authors to attribute multiple causes to the same mistake, highlighting the need for further research to improve algebra instruction (Pérez et al., 2019; Wicaksono et al., 2024).

Therefore, we consider that studying the errors students make when solving linear equations is essential for optimizing algebra instruction and strengthening mathematical learning. Identifying and analyzing these errors not only helps us understand the conceptual and procedural difficulties students face but also allows for the design of more effective teaching strategies to address these challenges. Additionally, detecting recurring error patterns enables teachers to anticipate potential difficulties and adjust their instruction to foster a deeper understanding of linear equations. Since this topic plays a key role in the transition from arithmetic to algebra and serves as the foundation for studying more advanced mathematical models, reducing the incidence of errors in solving linear equations significantly contributes to the development of mathematical thinking at higher levels. Along these lines, we agree with Pérez et al. (2019), who emphasize the need to investigate errors at post-compulsory education levels, as this will provide a more comprehensive view of the issue and allow for the design of effective teaching strategies for improvement.

Thus, this research aims to answer the following question: What errors do a group of Mexican High School students make when solving linear equations, and what are their causes? Consequently, our aim is to analyze these errors and propose explanations for their origin, based on the perspectives of Socas (1997) and Pérez et al. (2019).

Method

The theoretical references used in this research include the conceptualization of errors and their causes as proposed by Socas (1997), as well as the classification established by Pérez et al. (2019), which describes errors from the perspective of mathematical content. These aspects are detailed below.

Conceptualization of errors and their causes

According to Socas (1997), an error is understood as the visible manifestation of a difficulty in the learning process, acting as a barrier that prevents or hinders the achievement of educational objectives related to mathematical content. Additionally, various variables influence the teaching-learning process, making it difficult to identify the underlying causes of errors. However, three interrelated dimensions allow for the classification of student errors based on three distinct origins: obstacles (epistemological, cognitive, and didactic), lack of meaning, and affective-emotional factors. Given that our study focuses on students' responses and reasoning, epistemological obstacles will not be considered.

According to Palarea and Socas (1994), cognitive obstacles refer to knowledge that was once useful for solving certain problems but becomes rigidly ingrained in students' minds, making it difficult to adapt when facing new problems. On the other hand, didactic obstacles originate from the teaching process experienced by students and are linked to specific characteristics of the educational system, such as teaching methodologies, curricular organization, or the partial meaning conveyed at a given moment. These obstacles arise from the structure and delivery of education and can hinder the proper internalization and application of knowledge.

Errors related to the lack of meaning in algebra learning can arise in three stages of development—semiotic, structural, and autonomous—and manifest in three main ways (Ruano et al., 2008; Socas, 2007).

- The first stage refers to errors stemming from poor assimilation of arithmetic concepts, as the initial steps in algebra rely on manipulating basic notions such as arithmetic operations, negative numbers, and equality relationships.
- The second type of error is associated with difficulties in manipulating algebraic language, where students struggle to interpret algebraic symbols, such as letters representing unknowns and the equal sign, which require new meanings.
- The third type of error occurs when students misapply memorized formulas or procedural rules, using techniques without understanding their underlying principles or applying them in inappropriate contexts.

These errors reflect a lack of comprehension at different levels of algebra learning, ranging from symbolic representation to more autonomous and deeper understanding. Errors caused by affective and emotional factors often stem from blockages or memory lapses due to lack of concentration or from the use of incorrect techniques to avoid confronting concepts that generate insecurity, such as operations with negative numbers (Pérez et al., 2019).

Finally, according to the theoretical framework used in this study, a single error may have multiple underlying causes. The main contribution of this research is to explore the various possible causes of errors in each student, aiming to provide solid arguments for interpreting the mistakes made when solving linear equations.

Classification of Errors in Solving Linear Equations

For the purposes of this study, we adopt the classification of errors in solving linear equations proposed by Pérez et al. (2019), who distinguish two main types of errors: arithmetic errors and algebraic errors specific to equations. Below is a detailed description of this classification.

Arithmetic errors

Within this category, four types of errors are identified:

1. **Errors in integer operations.** These errors involve mistakes in addition, subtraction, multiplication, and division of integers while solving equations (Abrate et al., 2006).
2. **Errors in fraction operations**. These include mistakes when dividing a fraction by a number, multiplying and dividing fractions, or finding the least common multiple (Egodawatte, 2011).
3. **Errors in applying distributive property**. These errors occur when students incorrectly distribute a coefficient, typically multiplying it only by the first term inside the parentheses instead of all terms (Egodawatte, 2011).
4. **Errors in transposing terms across equality signs**. These errors happen when students incorrectly invert the fraction when moving terms across the equal sign (Pérez et al., 2019).

Algebraic errors specific to equations

This category is divided into conceptual errors and procedural errors.

Conceptual Errors

1. Failure to distinguish between a term with a variable and a constant term. The student incorrectly adds a term containing a variable to one that does not (Egodawatte, 2011).
2. Errors in handling coefficients. These occur when students transpose the coefficient of the variable incorrectly, treating it as if it were being added instead of multiplied.

Procedural Errors

Two types of procedural errors are closely linked to the solution method used by students (Kieran & Filloy, 1989):

1. Errors in the balance method. The balance method consists of performing the same operation on both sides of the equation. However, students often mistakenly apply the operation to only one side, disrupting equality.
2. Errors in the step-by-step rule method. The step-by-step rule method follows the principle that "what is added moves to the other side as a subtraction" and "what is multiplied moves as a division." Errors arise when students apply these rules incorrectly.

Additionally, several authors have noted that students tend to mix both methods, leading to the creation of new incorrect rules (Abrate et al., 2006; Hall, 2002). For example, a student might apply the multiplication rule but invert the sign incorrectly.

3. Errors in the order of operations. These occur when students do not follow the correct sequence of transpositions in solving the equation.

This research follows a qualitative case study approach. This methodology allows for a focus on meaning-making and the understanding of mathematical concepts within individuals' knowledge, where the final outcome is descriptive (Merriam & Tisdell, 2016). Additionally, it involves an inquiry process characterized by a detailed, comprehensive, and systematic analysis of the studied phenomenon (Rodríguez et al., 1999).

Research context and participants

The study was conducted in February 2025 at a public High School located in southern of Mexico. A total of 30 students (18 males and 12 females) voluntarily participated, all aged 17 to 18 years old and legally enrolled in the 12th grade. The participants reported having prior experience in solving linear equations, and most of them identified as competent in this skill. In fact, many expressed confidence in their ability to solve these types of equations and claimed to have obtained good results in previous evaluations. Hereafter, the study participants will be referred to as E1, E2, E3, …, E30.

Data collection

A Task-Based Interview (TBI) was designed and implemented following the principles established by Goldin (2000). This type of interview is characterized by minimal interaction between the interviewee —who solves the task— and the interviewer, represented by the researchers. The purpose is to focus on one or several pre-designed tasks, which may include questions, problems, or activities. This approach combines the interview —allowing for the exploration of the participant's thought processes through verbalizations— with an instrument (tasks) that facilitates the identification of the procedures and concepts used by the interviewee when addressing the proposed challenges.

A seven-item task was developed and validated by experts to ensure its difficulty level and relevance to the study's objectives. A pilot test was then conducted with two NMS students to verify the accessibility of the tasks and their alignment with the study's purpose. The pilot test lasted approximately 60 minutes and was video-recorded for later analysis. Based on the results, adjustments were made to two items in the task, which were then included in the final instrument used in the study.

The final instrument consisted of the task and its items, detailed in Table 1, specifically designed to identify the types of errors made by NMS students when solving linear equations, as well as their possible causes.

The interview protocol included general questions applicable to all items, such as:

- *"What does this term mean?"*
- *"Can you explain more about what you mentioned?"*
- *"Can you try a different approach?"*

Additionally, specific questions were included to further explore the participant's reasoning and knowledge regarding the solution of linear equations. These were used when the interviewee's responses were ambiguous, lacked clear reasoning, or when the aim was to encourage the exploration of alternative solutions.

Table 1. Task Presented to Students

Task: Solve the following equations and explain your procedure.
a. $4(-3x+8)=-5x$
b. $-3(2x-1)=4$
c. $-6x=24$
d. $-5x-8=-28$
e. $4x=\frac{1}{3}$
f. $\frac{x}{4}+\frac{x}{3}=2$
g. $\frac{x}{5}+6x=1$

Data Analysis

The interviews provided both written and verbal evidence from the participants, recorded through worksheets and video recordings. For the analysis of this data, the study employed the classification of error types proposed by Pérez et al. (2019) and the causal framework described by Socas (1997).

All researchers participated in this process. Each researcher individually reviewed the interviews and identified the errors committed by students along with their possible causes. Subsequently, the researchers convened to compare and discuss their findings. In cases of disagreement, they debated until reaching a consensus, ensuring rigor in error identification and minimizing bias from individual judgment.

Results and discussion

The analysis of students' responses when solving linear equations revealed a low percentage of correct solutions, suggesting that difficulties with this topic are not exclusive to high school students (Hall, 2002; Pérez et al., 2019; Kieran, 1989), but rather persist even at the NMS level. While addressing each linear equation, students made various types of errors. Table 2 presents the categories of errors along with the percentage of students who committed them.

Table 2. Errors Committed by Students

Type	Categories	Percentage of Students
Arithmetic Errors	Errors in integer operations	30%
	Errors in fraction operations	80%
	Error in the distributive property	50%
	Error in the inversion of equation members	40%
Algebraic Errors	Conceptual errors	80%
	Procedural errors	60%

Arithmetic Errors and Their Causes

During the process of solving linear equations, the case studies exhibited **arithmetic errors** in four distinct ways. Below, these errors are detailed, supported by evidence of their occurrence, and analyzed according to the framework proposed by Socas (1997).

Errors in Integer Operations

This type of error was observed in 30% of participants while solving equation (d), particularly when adding integers. While students correctly moved the constant to the other side of the equation by changing its sign, they made mistakes in the actual operation. Specifically, they added the values without applying the correct sign rules, often assigning the sign of the first number to the result (see extract from dialogue with E1 and Figure 1).

E1: Well, in the equation in part (d), what I do is solve for the x.

Researcher: Could you explain how you did it?

E1: Of course, the eight is subtracting and adding, so I have negative five x equals negative twenty-eight plus eight, then I add twenty-eight plus eight and it's thirty-six, and the result is negative thirty-six. Then, negative thirty-six divided by negative five gives us negative seven point two [performs the division].

Researcher: Do you always add whole numbers that way?

E1: Yes.

Interviewer: Okay, now I'll give you some operations to solve: $-4 + 5$; $-3 - 2$; $(-2)(-3)$; $(-4)/(-2)$.

E1: Sure! The first is -9, the second is -1, the third is -6, and the last is -2.

Interviewer: Was anything particularly difficult for you when solving the equation?

E1: Well, I've always struggled with operations involving signed numbers, and I get a bit nervous when solving them. I usually just do whatever seems right at the moment.

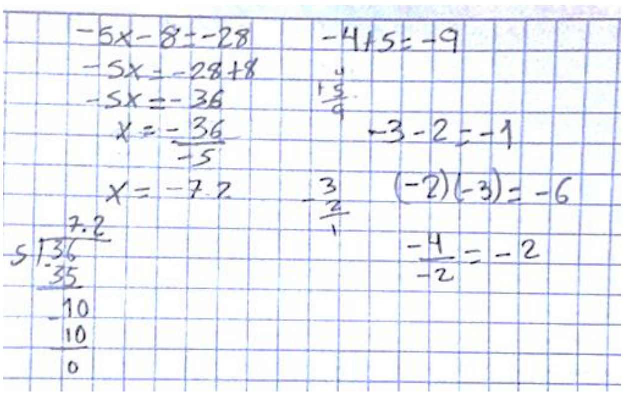

Figure 1. Error in Integer Operations Evidenced by E1 for Equation (d)

Based on students' written responses and verbal explanations during the interview, these errors appear to stem from a weak understanding of arithmetic concepts. Additionally, the interviews revealed that these errors were also influenced by affective and emotional factors, especially when dealing with signed numbers. Students frequently forgot basic rules or used improvised methods, likely due to insecurity when working with integers.

Errors in Fraction Operations

Another commonly identified error—committed by 80% of participants—was related to fractions when solving linear equations. These errors were evident in equations (e) and (f). Students struggled particularly when dividing a fraction by a whole number and when adding, subtracting, multiplying, and dividing fractions. In some cases, students who solved the equation in part f) added both the numerators and denominators, while others added the numerators and multiplied the denominators. In the case of part e), when dividing one-third by four, some students concluded that the result was four-thirds.

The Task-Based Interview (TBI) was crucial in identifying the cause of these errors. Auxiliary questions revealed that these mistakes originated from a lack of proper assimilation of arithmetic concepts. Additionally, students expressed that simply working with fractions increased their anxiety and perceived difficulty, revealing that affective and emotional factors played a role in these errors (see Figure 2).

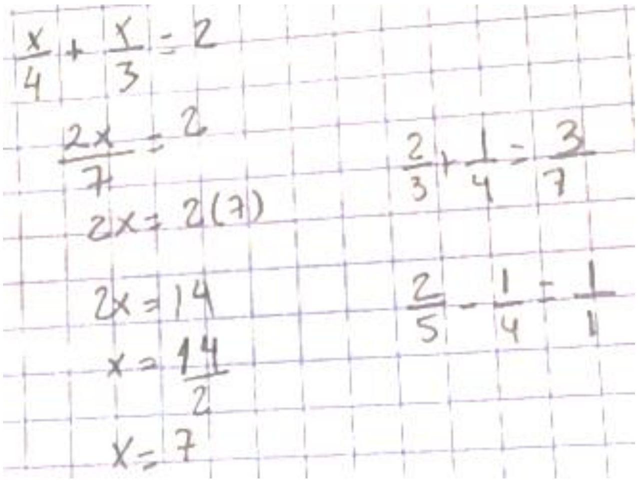

Figure 2. Error in Fraction Operations Evidenced by E4 for Equation (f)

Error in the Distributive Property

This error was identified in 50% of participants, particularly in equations (a) and (b). The most common mistake was an incomplete application of the distributive property, where students multiplied only the first term inside the parentheses, omitting the second.

During the TBI, students were also presented with an equation where the multiplying term was on the right side of the parentheses, and they repeated the same mistake. Additionally, some students did not recognize the need to apply the distributive property, while others forgot to account for signs when doing so. When faced with similar problems involving only numbers, they made the same mistakes as when working with algebraic expressions. This suggests that the root of this error lies in a lack of conceptual understanding of arithmetic operations.

Furthermore, during the interview, students who made this error mentioned that they did not remember how to solve these types of equations, which made them feel nervous and mentally blocked. This suggests that emotional and affective factors also contribute to this error.

E8: In b), minus three times two x's, we have minus six x's minus one equals four [writes -3$(2x - 1) = 4 \rightarrow -6x - 1 = 4$]. Then, I add 1 [writes $-6x = 4 + 1 = 5$], and finally, x is -5/6 [writes $x = -5/6$].

Interviewer: How would you solve the equation $(x + 4)2 = 1$?

E8: It's similar [writes $(x + 4)2 = 1 \rightarrow x + 8 = 1 \rightarrow x = 1 - 8 \rightarrow x = -7$].

Interviewer: Now solve $5(3 + 4)$.

E8: Sure [writes $5(3 + 4) = 15 + 4 = 19$].

Interviewer: Was anything particularly difficult for you when solving the equation?

E8: Well, we didn't cover these types of equations much, so I get nervous and feel like I freeze up.

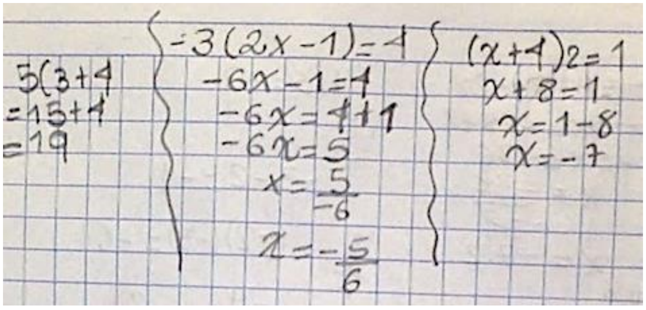

Figure 3. Error in the Distributive Property Evidenced by E8 for Equation (b)

Error in the Inversion of Equation Members

This error was observed in 40% of participants, indicating it is relatively common. It involves incorrectly inverting the numerator and denominator of a fraction in the final step, leading to an entirely different result.

This mistake typically occurred when the coefficient was greater than the independent term, causing students to misinterpret the division process. Many assumed that the larger number should always be divided by the smaller one, a misconception developed in early education when students are accustomed to dividing larger numbers by smaller ones.

E13: So, in (b), I isolate x [writes $-3(2x - 1) = 4 \rightarrow -6x + 3 = 4 \rightarrow -6x = 4 - 3 = 1 \rightarrow x = -6/1 = -6$].

Interviewer: Why did you divide six by one?

E13: So that the division works.

Algebraic Errors in Equations and Their Causes

Another type of error observed in the case of study participants' work is related to algebraic errors specific to linear equations. These errors emerged from two distinct perspectives: conceptual errors and procedural errors. Below, we detail how these errors arose and the reasoning that allowed us to identify their causes.

Conceptual Errors

These errors appeared in the responses of 80% of the case study participants and were classified into two subcategories. The first refers to the separation between the variable and the constant term, where students mistakenly combine a term containing a variable with one that does not (see excerpt from the dialogue with E30). The second type of error involves coefficient transposition, in which the coefficient of the variable is moved to the other side of the equation as if it were being added or subtracted rather than multiplied.

The error related to the separation of the variable and the constant term was the most common among the participants. This mistake occurs because students have not yet internalized the distinction between terms that contain variables and those that do not. According to Egodawatte (2011), this error arises because students attempt to simplify equations they perceive as excessively complicated. However, our study confirms that this issue is rooted in the manipulation of algebraic language, which links it to a lack of conceptual understanding.

> E30: Let's look at equation (b) [writes $-3(2x-1) = 4$]. First, we distribute [writes $-6x + 3 = 4 \longrightarrow -3x = 4 \longrightarrow x = -4/3$].
>
> Researcher: Can you explain how you combined $-6x + 3$?
>
> E30: Yes, negative six plus three equals negative three, and then I add the *x*.

In addition to the above, the coefficient-related conceptual error appeared less frequently, primarily in equation (c). Students who made this mistake were able to distinguish between variable terms and constants, as they generally separated them on different sides of the equation. However, they failed to recognize that coefficients are multiplicative factors of the variable. Consequently, they treated coefficients as independent terms, moving them to the other side of the equation through addition or subtraction (see excerpt from the dialogue with E20).

The TBI indicated that this error is not linked to "transposition rules", as students explicitly stated that the coefficient was subtracting from the variable. Therefore, we attribute this error to a misunderstanding of algebraic notation and structure. The frequency of this error increased significantly when the coefficient was a fraction.

> E20: Well, equation (c) is easier. [Writes $-6x = 24$] Since negative six is subtracting, it moves to the other side as an addition. [Writes $x = 24 + 6 = 30$]. So, the answer is thirty.
>
> Researcher: What operation is the coefficient performing on x?
>
> E20: Since it has a negative sign, it is subtracting.

Procedural Errors

Two types of errors were found in the procedural resolution of equations, appearing in 60% of the case study responses: errors in maintaining equality between both sides of the equation and errors in applying transposition rules.

Equality errors were the most frequent. This mistake involved an incorrect application of the balance method, where students performed an operation on only one side of the equation (see Figure 4). This occurs because students are generally taught to solve equations using the transposition rules and only resort to the balance method when the equation includes a negative coefficient, which they eliminate by multiplying by -1.

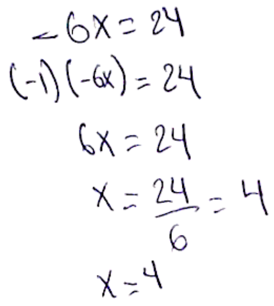

Figure 4. Incorrect application of the balance method by E25 in equation (c)

Based on our findings, we consider this a didactic obstacle, as its origin lies in the teaching process—specifically, in the limited use of the balance method by instructors. This error cannot be attributed to cognitive difficulty, as it results from a specific rule within this instructional approach: when the coefficient of *x* is negative, both sides of the equation must be multiplied by -1.

Another frequent error was the misapplication of transposition rules. This occurs when students fail to apply them correctly, either due to confusion or because they create incorrect rules. Those who struggle with these rules often transpose terms incorrectly, performing the same operation on the second member that was originally applied to the first. For example, if a term was multiplying on the left side of the equation, they move it to the right side while still multiplying.

The TBI provided evidence that this error arises from a failure to properly analyze the operation applied to the transposed term. Thus, the root cause lies in the misapplication of procedural rules, which is linked to a lack of conceptual understanding. We also say that this issue may be influenced by the instructional method used. Kieran (1980) suggests that students who work with the balance method are more aware of the operation each term performs compared to those who rely solely on transposition rules.

In contrast, some students created their own incorrect rules, blending different transposition methods. For instance, as shown in Figure 5, they divided what was originally multiplying but also changed its sign.

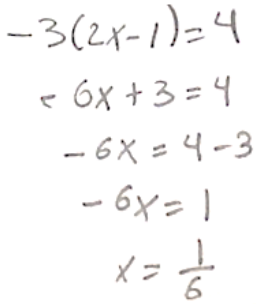

Figure 5. Incorrect transposition rule applied by E27 in equation (b)

This type of procedural error had already been identified in previous studies (Abrate et al., 2006; Castellanos & Moreno, 1997; Hall, 2002). In our study, it only occurred when the coefficients were negative numbers, specifically in equations (b), (c), and (d), a finding consistent with Pérez et al. (2019) in secondary school students. This conclusion was reinforced through the Think-Aloud Protocol, where students repeatedly expressed confusion about why their solution was incorrect, revealing difficulties in working with negative numbers. As a result, they created alternative rules to avoid dealing with these values. We interpret this as a manifestation of a lack of conceptual understanding, leading to the misapplication of algebraic procedures.

Another type of error observed was related to the order of operations. This mistake involved performing transpositions in an incorrect sequence. The Think-Aloud Protocol indicated that the cause of this error is that students have not fully grasped that when transposing a divisor, it must divide all terms in the given expression (see excerpt from the dialogue with E3). We interpret this error as a case where a previously effective strategy becomes difficult to adapt to a new context, making it a cognitive obstacle.

> E3: In equation (g) [writes $x/5 + 6x = 1 \longrightarrow x + 6x = 1(5) \rightarrow 7x = 5 \rightarrow x = 5/7$], first, the five is dividing, so I move it to multiply the one; then, I add one *x* plus six *x* to get seven *x* equals five; finally, I divide by seven.
>
> Researcher: Do you always solve this type of equation this way?
>
> E3: Yes.
>
> Researcher: What reasoning did you use?
>
> E3: Well, I first do multiplications or divisions, then additions and subtractions. That’s how I remember it from when I learned algebra.

Discussion

The results of this research are consistent with those reported by Pérez et al. (2019) in secondary school students, suggesting that difficulties in solving linear equations persist at the NMS level. This pattern indicates that students struggle to overcome these challenges, even after multiple opportunities to engage with the topic. A particularly relevant aspect identified is the increasing influence of affective and emotional factors, which significantly impact the learning process (Usán et al., 2019). Since students are aware that this topic has been addressed since secondary school, they recognize an expectation of mastery. However, when they fail to meet this expectation, they experience frustration and anxiety (Chevrier et al., 2019; Diego-Mantecón & Córdoba-Gómez, 2019). This awareness of their difficulties not only exacerbates their academic insecurity but also reinforces their negative perception of mathematics, potentially contributing to a vicious cycle of demotivation and poor performance (Gómez-Chacón, 2000; Medrano et al., 2016). Therefore, it is essential to address not only the cognitive aspects of teaching but also the emotional and affective factors that influence students' learning, fostering an environment that enables them to overcome these barriers (García-González et al., 2021).

Additionally, the majority of students made errors when performing operations with fractions, a finding that aligns with Cadenas (2007). This author observed that, when obtaining a fraction as a result of an operation, students often erroneously assume that the larger number should be placed in the numerator. This misconception may be partly explained by the challenges that teachers face when teaching the concept of fractions in their classrooms (Melquiades-Martínez et al., 2023; Putra, 2018). In fact, recent studies have revealed that, among teachers, procedural knowledge of fractions often outweighs conceptual understanding (Barthi, 2022; Castro-Rodríguez & Rico, 2021). When analyzing the representations used in classrooms to teach fractions, it has been observed that teachers frequently rely on a limited set of models, particularly the area model using circles, which contributes to students' difficulties in working with fractions (Alqahtani et al., 2022).

Beyond arithmetic challenges, students also displayed both conceptual and procedural algebraic errors—findings similar to those reported by Adu et al. (2015) with Ghanaian students. This may be explained by the fact that, in the school context, the instruction of linear equations primarily emphasizes procedural techniques, particularly the transposition rules (Pérez et al., 2019). Students tend to memorize these rules to solve equations, which may create the illusion of success when they successfully isolate the variable and obtain a solution. However, in the long term, these rules are often forgotten or even distorted when students attempt to apply them in new contexts (Chazana et al., 2008).

The findings of this research indicate that students' errors in solving linear equations partially stem from gaps in their prior knowledge, including operations with integers, operations with fractions, the distributive property of multiplication over addition, and even the addition and subtraction of like algebraic terms (Amaya-García, 2022). This lack of consolidation limits their learning and hinders their ability to grasp more advanced concepts, such as solving linear equations.

The results of this study align with previous research conducted across various student populations at the basic and secondary levels, highlighting the limited impact of formal instruction on students' comprehension of this concept (García-García, 2024).

Conclusions

The findings of this study reveal that NMS (High School Level) students make arithmetic errors when solving linear equations. These include difficulties in performing operations with integers, working with fractions, applying the distributive property, and incorrectly rearranging the terms of an equation. Through the analysis of students' reasoning, it was inferred that the main causes of these errors stem not only from a weak assimilation of arithmetic concepts and cognitive obstacles, but also, and significantly, from affective and emotional factors. Emotional aspects—such as anxiety, frustration, and lack of confidence—were shown to play a crucial role in problem-solving processes, negatively impacting students' ability to correctly apply learned procedures.

Additionally, both procedural and conceptual algebraic errors were identified. According to the theoretical framework, these errors arise from limited manipulation of algebraic language, incorrect application of procedural rules, and the presence of both didactic and cognitive obstacles. Therefore, the emotional dimension must be considered a key factor in understanding and addressing the difficulties students face when working with linear equations.

It is crucial for future research to propose specific didactic approaches for teaching linear equations, considering the errors and underlying causes documented so far. Such efforts would be instrumental in significantly improving students' learning processes and fostering a deeper conceptual understanding of algebra.

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