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The Extended Theory of Connections (ETC) and its Contributions to Mathematics Education: Some Reflections on the Path Taken

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Abstract

For several years, a fundamental idea has been promoted that establishing mathematical connections is important for understanding mathematical concepts. This statement has led many researchers to consider that students' difficulties are caused by errors in problem-solving that arise from failing to make necessary connections linked to procedures, graphic representations, meanings, among other aspects, that are inconsistent with institutional mathematics. Given this problem, the Extended Theory of Connections emerged through consensus in the literature, importance in curricular organizations, the quality of analyses of mathematical activity, and its theoretical and methodological development with intra-mathematical, extra-mathematical, ethnomathematical, and neuro-mathematical connections. Specifically, this article addresses a review of the literature on connections that contains a theoretical framework, an appropriate methodology for identifying connections, and some practical cases of connections. It is asserted that mathematical connections are a topic of interest and should continue to be promoted in classrooms to improve the teaching and learning processes of mathematics by involving sociocultural and neurocognitive aspects.

Resumen

Hace varios años se ha promovido una idea fundamental referida a que el establecimiento de conexiones matemáticas es importante para la comprensión de los conceptos matemáticos. Con esta frase se ha concientizado a muchos investigadores para que consideren que las dificultades de los estudiantes son ocasionadas por los errores en la resolución de problemas que emergen por no hacer conexiones necesarias ligadas a procedimientos, representaciones gráficas, significados, entre otros aspectos, que no son consistentes con las matemáticas institucionales. Dada esta problemática, surge la Teoría Ampliada de las Conexiones por medio de consensos de la literatura, importancia en los organismos curriculares, calidad de los análisis de la actividad matemática y su desarrollo teórico y metodológico con las conexiones intramatemáticas, extramatemáticas, etnomatemáticas y neuromatemáticas. Particularmente, en este artículo se aborda una revisión de la literatura sobre conexiones que contiene marco teórico, metodología adecuada para identificar conexiones y algunos casos prácticos de conexiones. Se asegura que las conexiones matemáticas son un tema de interés y que se deben seguir promoviendo en las aulas de clases para mejorar los procesos de enseñanza y aprendizaje de las matemáticas involucrando aspectos socioculturales y neurocognitivos.

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Introduction

The scientific community recognizes that mathematical connections are essential in education and daily life, enabling people to apply math to real-world problems (Berry & Nyman, 2003; Rodríguez-Nieto & Font, 2025). These links foster logical and critical thinking by connecting abstract concepts to real experiences, such as calculating finances or interpreting data. They also support functional understanding and informed decision-making in various contexts (Rodríguez-Nieto, 2021a; Son, 2022). Moreover, in everyday practices, people relate empirical knowledge to institutional mathematics, showing that connections are not just didactic tools but means to understand the world (Campo-Meneses & García-García, 2023; García-García, 2024; Rodríguez-Nieto et al., 2025a).

Various authors and curricular entities agree that mathematical connections play a fundamental role in helping students understand mathematical concepts throughout their educational career (Koestler et al., 2013; MEN, 2006; NCTM, 2000). These connections facilitate the link between meanings, representations, statements, everyday situations, and formalized mathematics (García-García & Dolores-Flores, 2018; Hiebert & Carpenter, 1992; Rodríguez-Nieto, 2020; Rodríguez-Nieto et al., 2021a; Rodríguez-Nieto et al., 2021b; Wittmann, 2021).

Pambudi et al. (2018) highlight that mathematical connections are key to problem-solving success. Activating specific connections is crucial and can be observed in students' actions, verbal, or gestural arguments during tasks (García-García & Dolores-Flores, 2018), and are also present in the curriculum. In multiple curricular systems internationally, mathematical connections are considered essential. They are recognized as a key competency for problem-solving and for establishing relationships between prior ideas and new knowledge. This is evident in the secondary school curriculum of Catalonia (Departament d’Ensenyament, 2017), in the case of Colombia (MEN, 2006), Singapore (MOE, 2006), South Africa (Mwakapenda, 2008), the United States (NCTM, 2000), in Mexico Autonomous University of Guerrero [UAGro], 2020), and Brazil (São Paulo State, 2012).

Mathematical connections are key in both curricula and educational research. Rodríguez-Nieto (2020) analyzes internal, external, and meaning connections in everyday measurement systems from an ethnomathematical lens, while Rodríguez-Nieto (2021b) defines the ethnomathematical connection as the link between daily mathematical practices and formal concepts. Furthermore, different studies have proposed typologies of mathematical connections, developed from the analysis of both qualitative and quantitative data, with special emphasis on thematic analysis. For example, in an analysis carried out in SCOPUS, connections were identified as a highly researched topic, and authors such as García-García (2024) and Rodríguez-Nieto and Font (2025) are increasingly promoting analyses to better understand mathematical concepts (see Figure 1).

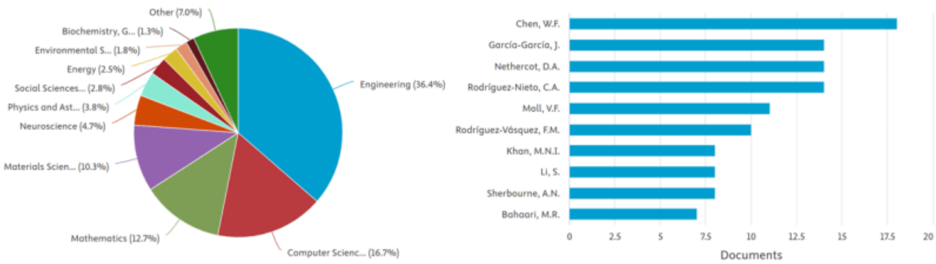


Figure 1. Authors and areas of study where mathematical connections influence.

Furthermore, these investigations on mathematical connections have been published in relevant journals in Mathematics Education and in other fields such as engineering, computing, physics, neuroscience, among others (Figure 2).

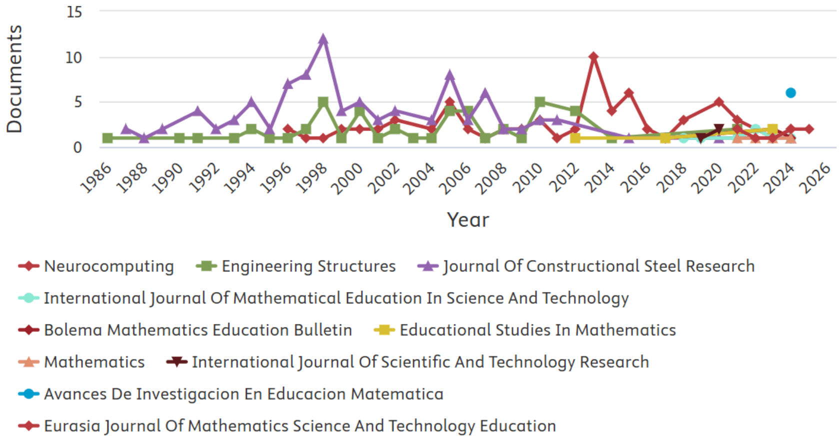


Figure 2. Journals where works on connections have been published.

Other searches found research on mathematical connections in Google Scholar in both Spanish and English (see Figure 3).

Figure 3. Research on connections found in Google Scholar.

Specifically, this article *addresses a review of the literature on connections that contains a theoretical framework, an appropriate methodology for identifying connections, and some practical cases of connections*.

Methodology

In studies on mathematical connections, qualitative methodologies have been used primarily, following the approach of Cohen et al. (2018), to describe and analyze the mathematical activity of students, teachers, and people who are not necessarily mathematicians, such as artisans, farmers, etc. In this context, the exploration of connections is interpretive in nature, assuming a didactic and pedagogical stance based on consolidated approaches, for example, the ETC. It should be noted that this article has the air of a literature review as carried out in Rodríguez-Nieto et al. (2022c). This is a suitable methodology for identifying connections. A particular case of a study of mathematical connections was carried out by Rodríguez-Nieto (2021b) in his doctoral thesis entitled in its original Spanish version: Analysis of mathematical connections in the teaching and learning of derivatives based on a networking of theories between the Theory of Connections and the Onto-semiotic Approach; where the following research questions were answered:

Main question (MQ) (Rodríguez-Nieto, 2021b):

> *How does the application of onto-semiotic constructs complement the analysis, from a cognitive perspective, of the mathematical connection processes necessary to solve derivative problems?*

This question is specified in the following specific questions (SQ): How can the combined use of the Extended Theory of Connections (ETC) and the Onto-semiotic Approach (OSA) help characterize, expand, and analyze the different categories of mathematical connections, and explain both the concordances between these theories and the difficulties students face in establishing key connections when solving derivative problems?

A plausible methodological scheme for developing this work is presented in Figure 8, where two methods were addressed, such as thematic analysis and onto-semiotic analysis, but they were enhanced through networking between theories between ETC and OSA (Figure 4).

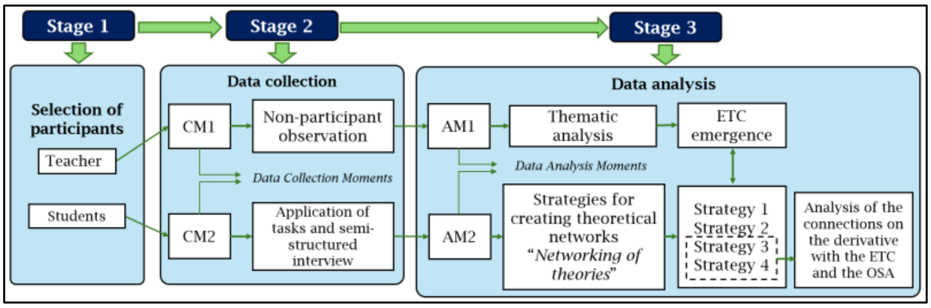


Figure 4. Methodological structure of Rodríguez-Nieto's doctoral thesis (2021b).

The methodological approach followed in this research is essential to ensuring a rigorous analysis consistent with the stated objectives, as it allows for an in-depth understanding of the complexity of the phenomenon studied: the mathematical connections surrounding the concept of derivative (Rodríguez-Nieto, 2021b). In this regard, in the first stage, the study participants were selected, which included a teacher and his group of students.

Subsequently, in the second stage, data collection was carried out using two methods applied at two different times. In the first stage of methodological construction (CM1), non-participant observations were conducted in seven classes in which the teacher interacted with his students around the concept of derivative. In the second stage (CM2), semi-structured interviews were conducted with the students while they completed tasks related to this concept. The third stage corresponded to data analysis, also structured into two analytical moments (AM1 and AM2), corresponding to the previous collection moments (CM1 and CM2). Thus, to answer the first research question, focused on the emergence of ETC, a deductive or theory-based thematic analysis was applied in the first phase of analysis (AM1). The initial categories were those proposed in Businskas's (2008) model, complemented by contributions from other researchers (nine types of mathematical connections defined a priori).

To address the second research question, the second phase of analysis (AM2) used the theoretical networking strategies developed by Prediger et al. (2008) to articulate ETC with the Onto-semiotic Approach to Mathematical Knowledge and Instruction (OSA). Finally, to answer the third and fourth questions, the thematic analysis suggested by ETC was used in conjunction with OSA for the analysis of mathematical activity, which allowed for a more precise characterization of the emerging mathematical connections around the concept of derivative. Now, in terms of data collection, the most recommended approaches from a connection theory perspective are semi-structured or task-based interviews and classroom observations (participant or non-participant), where rich evidence such as high-quality photographs, videos, and field notes are obtained. Data analyses are then conducted from two perspectives: thematic analysis (Braun & Clarke, 2006) and onto-semiotic analysis (Rodríguez-Nieto et al., 2021a, 2022a, 2023b; 2023b, 2024, 2025a).

Along these lines, Rodríguez-Nieto et al. (2021a) conducted a deductive thematic analysis of the mathematical activity of pre-service mathematics teachers using a priori connection categories. It was successful because the nuances and details of the codes and themes were clear and provided conclusive evidence.

Description of the phases of thematic analysis

The following phases are, metaphorically speaking, the backbone of data analysis according to Braun and Clarke (2006).

Phase One: Data Familiarization

The information obtained from the interviews was transcribed and converted into text, with the goal of gaining familiarity with the data.

Phase Two: Generation of Initial Codes

Words and phrases in the data that indicated some type of mathematical connection were identified, which allowed for the generation of initial codes.

Phase Three: Search for Subthemes and Themes

Based on codes with similar characteristics, subthemes and themes were identified. In this research, the themes correspond to the types of mathematical connections a priori described in the conceptual framework.

Phase Four: Review of Subthemes and Themes

The subthemes and themes identified in the previous phase were reviewed and triangulated by experts. This triangulation allowed for the elimination or formation of new themes from the data.

Phase Five: Definition and Naming of Themes

The identified themes, along with their common characteristics, were named. In this phase, the researchers reached a consensus on the types of mathematical connections identified in the data.

Phase Six: Report Preparation

A results report was prepared showing the themes (mathematical connections) found throughout the analysis.

Now, the other data analysis is carried out from an onto-semiotic perspective, assuming seven phases to also assess neuro-mathematical connections.

Connection analysis taking into account ETC + OSA + neuromathematics

Phase 1: Transcription of interviews/observations or organization of written productions

The written productions of the students and/or teachers are organized. The researchers familiarized themselves with the participants' responses. This phase is essential to ensure a thorough reading, analysis, and understanding of the information collected.

Phase 2: Temporal narrative

The student or teacher's problem-solving is explained mathematically. This narrative includes the mathematical practices performed and some important primary objects, considered protagonists of the mathematical activity. The narrative is used to analyze the mathematical and neuro-mathematical connections.

Phase 3: Mathematical practice

Mathematical practices are described as a sequence of actions regulated by institutional norms useful for problem-solving. These practices reveal the basis of each connection, both in the brain (neuromathematics) and on paper (through ETC articulated with the OSA approach). The identification of these practices depends on the coding process (c), that is, the raw data obtained from the transcripts of the participants' observations.

Phase 4: Cognitive Configuration

This is the system of primary mathematical objects that a subject mobilizes as part of the mathematical practices developed to solve a problem. These objects are a fundamental part of the connection, as they usually represent the beginning (antecedent) and the end (consequent) of its structure.

Phase 5: Semiotic Functions (SFs)

Semiotic functions are established between the primary objects of the cognitive configuration. In this way, the mathematical connections proposed by the ETC are formed and visualized.

Phase 6: Mathematical Connections

Semiotic practices, processes, objects, and functions are integrated and recorded in a table that allows the mathematical connection to be originated and detailed.

Phase 7: Neuro-mathematical Connections

Each of the mathematical practices carries implicit neuro-mathematical connections, which relate brain areas before the mathematical connections are visualized or externalized.

Results

Mathematical connections across educational levels

Mathematical connections play an essential role at all levels of education, established by institutions around the world. At the primary education level, it is considered fundamental for students to develop these connections. Frías and Castro (2007) explored how connections influence the symbolic representation of two-step arithmetic problems among fifth and sixth grade students. Their research showed that links or nodes have an important impact on the resolution of these types of problems, since it is usually complicated to convert a verbal statement into a mathematical expression. Specifically, Sofía had 25 candies. Her friend Lucía gave her 18 more. Sofía then gave 12 candies to her brother. How many candies does Sofía have now? To solve this problem, a student could use two stages or procedural mathematical connections connected to problems with semantic structures of change increase and change decrease:

First stage - Addition (what you receive): $25 + 18 = 43$

Sofia now has 43 candies after Lucía gave her more.

Second stage - Subtraction (what you give away): $43 - 12 = 31$

After giving away 12, he has 31 candies left.

Final answer: Sofia now has 31 candies.

In this context, Rodríguez-Nieto et al. (2025b; 2025c) emphasize that fostering these connections from the early school years with additions and subtractions can lay the foundation for deeper mathematical understanding at later levels. Barmby et al. (2009) proposed a representational reasoning model, focusing on the matrix representation of multiplication. They defined understanding as the creation of links between mental representations of mathematical concepts, with reasoning being the process that connects these parts. Kayhan et al. (2017) found that eighth-grade students fail to make meaningful connections between mathematics and everyday life, limiting themselves to numbers and shapes. Safi and Desai (2017) emphasized the importance of using algebraic and geometric representations and highlighted how manipulative tools (2D and 3D blocks) can strengthen understanding through multiple representations. Lianawati and Purwasih (2018), for their part, linked the ability to make mathematical connections with sociocultural construction, observing gender differences: while boys tend to make better conceptual connections, girls are influenced by social roles and cultural factors.

In Mexico, relevant research on mathematical connections has been conducted. García-García and Dolores-Flores (2018) identified 223 mathematical connections in calculus problem-solving, classified by representation type and purpose (procedural, meaningful, characteristic, and reversibility). Later, García-García and Dolores-Flores (2021) observed students graphing derivatives and antiderivatives through visualization. Dolores-Flores et al. (2019) found that many students failed to relate gradient to other concepts like velocity and acceleration. At the university level, Dolores-Flores and García-García (2017) analyzed students' intra- and extra-mathematical connections while solving contextual problems. Eli et al. (2011) highlighted the lack of connections between symbolic and graphical representations in teaching geometry, later concluding that curricular connections positively impacted teaching (Eli et al., 2013).

Other studies have highlighted cognitive difficulties experienced by prospective teachers in topics such as conics (Moon et al., 2013) or non-routine problem-solving (Rodríguez-Nieto et al., 2025c). Caviedes et al. (2019) noted that these same teachers preferred numerical procedures and formulas over intuitive geometric methods. Gil et al. (2019) categorized prospective teachers' connections into three types: superficial, procedural, and conceptual, noting an increase in the latter two. Yavuz-Mumcu (2018) concluded that many preservice teachers do not adequately understand or apply concepts such as derivatives in problem situations, which limits their ability to make connections between representations. Rodríguez-Nieto et al. (2021b) observed that, although preservice teachers made different types of connections, some showed limited understanding of derivatives, which affected their ability to correctly graph or formulate tangent equations.

It is essential for teachers to promote mathematical connections, as these allow students to relate mathematics to other disciplines and real-life situations (NCTM, 2000). Otherwise, teachers may be unprepared to guide their students in this key aspect of mathematical reasoning (Eli et al., 2013). The NCTM (2000) highlights that. In contexts such as units of measurement, conversions, or algebra, teachers must facilitate students' understanding of the relationship between formulas and real-world objects (p. 46). Businskas (2008) analyzed how secondary school teachers establish connections in quadratic problems, identifying five types: distinct representations, implication, part-whole, procedural, and instructional-oriented. Mhlolo (2012) proposed a tool to assess the quality of connections, ranging from no connections (level 0) to justified and coherent connections (level 2). Subsequent studies reported that most teachers reported low levels of connection. After reviewing research at all levels (Kenedi et al., 2019; Rodríguez-Nieto et al., 2021a), it is seen that various concepts are addressed such as basic operations (Frías & Castro, 2007), polynomial functions and rate of change (Dolores-Flores et al., 2021) or derivatives and integrals (García-García & Dolores-Flores, 2018; 2021). However, difficulties persist in relating mathematics to everyday life and connecting analytical representations with graphs, especially in the study of the concept of derivative. Pino-Fan et al. (2018) noted that some prospective teachers fail to integrate the various meanings of the derivative, such as its zero value when the tangent is horizontal.

Currently, ETC has been used in several works with different themes due to the problems associated with understanding mathematical concepts and also the assessment of everyday practices developed in diverse sociocultural environments. In 2024, two monographs published in Research in

Mathematics Education and Advances in Research in Mathematics Education addressed this topic from different theoretical and methodological perspectives. Among the most notable contributions are studies on connections in area measurement (Caviedes et al., 2024), the use of the Realization Tree Mediator as a training tool (Weingarden, 2024), curriculum analysis using the TIMSS model (Peters, 2024), and the role of multiplicative thinking in mathematical reasoning (Day et al., 2024). These works delve into how mathematical connections impact teaching, cognition, and curriculum design.

For its part, the AIEM journal compiled research focused on extra-mathematical connections and their educational potential (Font & Rodríguez-Nieto, 2024). Of note are the studies by Barragán-Mosso et al. (2024) on linear equations in high school, and by Hatisaru et al. (2024) on algebraic problem solving in future teachers. Vargas et al. (2024) identified metaphorical and interdisciplinary connections in teacher training students, and Ledezma et al. (2024) proposed a detailed classification of connections in modeling processes. Furthermore, they addressed perspectives such as ethnomathematics in cultural contexts (Manchego-Palacio et al., 2024), the analysis of classroom interaction (De Gamboa et al., 2014), and the treatment of the concept of vector in textbooks (Rodríguez-Vásquez et al., 2024). Together, these studies enrich the theoretical and practical landscape of mathematical connections, proposing analytical frameworks that favor more integrative and meaningful approaches to mathematics teaching.

Typologies of Mathematical Connections: Advances, Approaches and New Projections

Throughout the specialized literature, various studies have proposed models that typify mathematical connections, providing different perspectives on how they manifest in teaching, learning, and problem-solving. Research such as that by Evitts (2004), Businskas (2008), Lapp et al. (2010), Eli et al. (2011), and García-García and Dolores-Flores (2018, 2021) have presented relevant classifications that allow us to understand the variety and richness of connections that can be established in mathematical work. For example, Evitts (2004) identified five types of connections used by teachers when addressing curricular problems: modeling, structural, representation, procedural-conceptual, and between mathematical concepts. Businskas (2008) proposed instruction-oriented, procedural, part-whole, implication, and different representational connections. Lapp et al. (2010) highlighted computational, figurative, resource, and relational connections, while Eli et al. (2011) introduced categorical, procedural, feature-based, derivative, and curricular connections.

From another perspective, García-García and Dolores-Flores (2018, 2021) proposed a fundamental distinction between intra-mathematical and extra-mathematical connections, organizing their typology based on the type of problem that activates them. Their model, later expanded by Campo-Meneses and García-García (2023), points out that even intra-mathematical connections can emerge from application situations. Furthermore, other theoretical approaches have enriched the conceptualization of mathematical connections. Ethnomathematics has differentiated between internal and external connections based on their use in everyday measurement systems (Rodríguez-Nieto, 2020, 2021a). Connections have also been explored within the framework of theories such as conceptual metaphor (Lakoff & Núñez, 2000), the onto-semiotic approach (Godino et al., 2007, 2019), and studies on theoretical interconnection in mathematics education (Prediger et al., 2008; Radford, 2008).

Theoretical tools of the Extended Theory of Connections (ETC)

A significant development in this area is ETC, developed in close collaboration with the Onto-semiotic Approach to Mathematical Knowledge and Instruction (OSA). This theoretical development has been strengthened by collaborative networks between researchers from both perspectives, generating robust analytical tools that allow for a more complex view of mathematical connections.

From this perspective, a mathematical connection is understood as the visible manifestation of a conglomerate of practices, processes, objects, and semiotic functions that relate them (Rodríguez-Nieto et al., 2022a). These connections are key to mathematical understanding and creativity and are classified into two broad categories: intra-mathematical, which link concepts, procedures, or representations to each other, and extra-mathematical, which establish relationships between mathematical concepts and real-life situations or non-mathematical contexts. The ETC proposes a detailed typology that includes types of connections, each with specific functions within learning and problem-solving. Each of these connections is summarized in Figure 5.

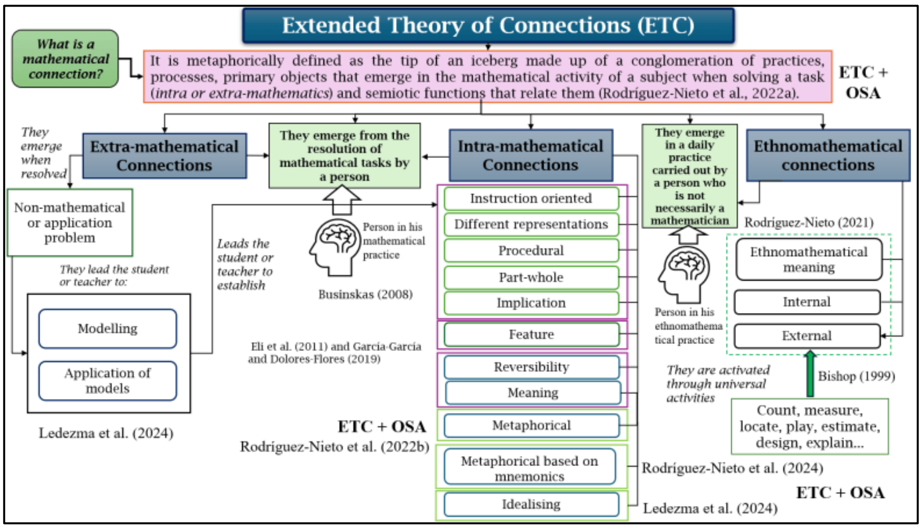


Figure 5. Synthesis of ETC (Rodríguez-Nieto et al., 2024).

1) Modelling: refers to the connection established between mathematics and the real world, or other sciences, through mathematical concepts and real-life tasks. It involves constructing a mathematical model to solve a real-world problem by using various knowledge (mathematical or not) and performing actions (algebraic, symbolic, graphic, etc.) to achieve a solution (Campo-Meneses & García-García, 2023; Dolores-Flores & García-García, 2017; Evitts, 2004).

2) Instruction-oriented: It refers to the understanding and application of a mathematical concept D derived from two or more related concepts, B and C. These connection types can be recognized in two forms: 1) the relationship of a new topic with previous knowledge, and 2) the mathematical concepts,

representations, and procedures connected are considered fundamental prerequisites that people must have to develop new content (Businskas, 2008).

3) Procedural: this connection is one of the most used by people and is evident when rules, algorithms, or formulas are used to arrive at a result (García-García, 2019). For example, to find the solutions of a quadratic equation $\boldsymbol{ax^2 + bx + c = 0}$ you can use the formula $\boldsymbol{x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}}$.

4) Part-whole: this connection is identified when it is institutionally assumed that A is a generalization of B, where B is a particular case of A. For example, the function $\boldsymbol{P(x) = 2x^3 - x^2 - 3x - 9}$ is a particular case of the general expression $\boldsymbol{f(x) = ax^3 + bx^2 + cx + d}$. These relationships can be of inclusion when a mathematical concept is contained in another (Businskas, 2008).

5) Implication: this connection is based on a logical relationship if-then $(P \rightarrow Q)$ (Businskas, 2008; Mhlolo, 2012). If a function $\boldsymbol{f}$ is increasing on an open interval (a, b), then $\boldsymbol{f'}$ is positive on that same interval.

6) Different representations: These mathematical connections can be alternate or equivalent. It is alternate if a person represents a mathematical concept in two or more different ways in different registers of representation: graph-algebraic, verbal-graph, etc. (Businskas, 2008). For example, an alternate representation is shown in Figure 6, where the vector $\vec{\boldsymbol{V}} = \boldsymbol{i - 3j + k}$ graphed. While an equivalent representation involves a transformation within the same register, such as algebraic to algebraic, graph to graph, or symbolic to symbolic. For example, $\vec{\boldsymbol{V}} = \boldsymbol{i - 3j + k}$ is equivalent to $\vec{\boldsymbol{V}} = \boldsymbol{\langle 1, -3, 1 \rangle}$ in the algebraic or symbolic semiotic register.

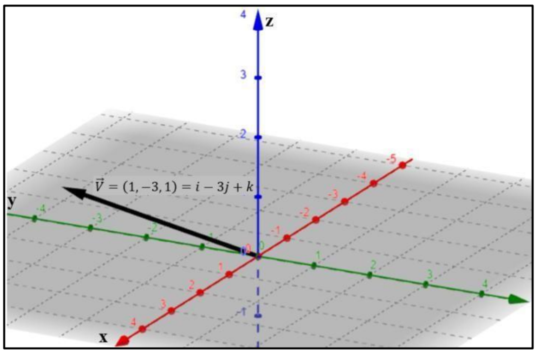


Figure 6. Connection between different representations (alternate) (Created in GeoGebra).

7) Feature: It is identified when the person manifests some characteristics of the concepts or describes its properties in terms of other concepts that make them different or similar to others (Eli et al., 2011; García-García & Dolores-Flores, 2021).

8) Meaning: this connection is activated "when students attribute a meaning to a mathematical concept as long as what it is for them (which makes it different from another) and what it represents; it can

include the definition that they have built for these concepts" (García-García, 2019, p. 131). Likewise, students or teachers express what the mathematical concept means to them, including their context of use or their definitions (García-García, 2019, p. 131).

9) Reversibility: It is present when a student or teacher starts from a concept P to get to a concept Q and invert the process starting from Q to return to P (García-García & Dolores-Flores, 2021). This connection is activated when the Fundamental Theorem of Calculus is used to link both concepts, and when the person establishes a bidirectional relationship between the derivative and the integral as operators.

10) Metaphorical: it is understood as the projection of the properties, characteristics, and other aspects of a known domain to structure a less familiar one. For instance, when a teacher or student uses verbal expressions like “travel through the graph without lifting the pencil from the paper”, they implicitly evoke the conceptual metaphor “the graph is a path” (Rodríguez-Nieto et al., 2022b).

11) Metaphorical connections based on mnemonics: this connection is “understood as the relationship established by the subject between a mnemonic rule (often a familiar resource) and a mathematical object, rule, or mathematical procedure to memorize and use strategically more easily” (Rodríguez-Nieto et al., 2024, p. 18). These types of connections are both inclusive and recursive, with three key elements to consider: a) keywords that are similar to the word (or term) being referenced; b) acronyms, which are formed when the first letter of each word in a list is used to construct a new word. c) acrostics which consist of constructing a sentence, where the first letter of each constitutes the term studied (Rodríguez-Nieto et al., 2024).

12) Idealising: this connection relates an ostensive to a non-ostensive. Its function is to dematerialize the ostensive and turn it into an ideal mathematical object (for example, the bottom of a tank rounded is considered circle/circumference) (Ledezma et al., 2024).

Cantillo-Rudas et al. (2024) extend the theory of mathematical connections, exploring neuro-mathematical links and brain activity during mathematical tasks. These developments address the multirepresentational and semiotic nature of mathematical knowledge, integrating cognitive, cultural, and contextual dimensions. Metaphorical connections project properties from familiar domains, while idealizing connections abstract concrete objects to ideal entities.

Furthermore, the projections and needs of ETC have been recognized from the analysis of the aforementioned models and the consolidation of ETC as an integrative framework. The following projections and needs can be established for research and teaching, where theory becomes practice:

- It is proposed to empirically expand typologies of mathematical connections by validating them across different domains and educational levels.

- Additionally, there is a need to theoretically articulate them with approaches such as ethnomathematics, mathematical neuroeducation, or semiotic theories. It is also necessary to develop assessment tools that identify and classify the connections mobilized during problem-solving. Finally, it is suggested that both initial and ongoing teacher training can focus on these connections to promote flexible thinking and enhance teaching through authentic situations, opening new possibilities for educational research in mathematics.
- Neuro-mathematical research: The emerging field of research on mathematical connections and brain activity (Cantillo-Rudas, 2025; Cantillo-Rudas et al., 2024) opens a field of exploration for understanding the cognitive and neurobiological dimensions of connections, which could revolutionize traditional pedagogical approaches.

The ETC is an integrative and evolutionary proposal with great potential to reorganize current understandings of connections and, in turn, to open new research and teaching avenues in mathematics education.

Synthesis of the ETC-OSA theoretical articulation

Establishing a theoretical network of theories between two or more theories represents a powerful strategy for exploring how their contributions can be coherently integrated, respecting the conceptual and methodological principles that underpin them. This strategy not only allows for the identification of convergences or tensions between theoretical frameworks but also contributes to a deeper understanding of the inherent complexity of the phenomena involved in the teaching and learning processes of mathematics (Kidron & Bikner-Ahsbahs, 2015; Prediger et al., 2008). Along these lines, this research builds on the work of Rodríguez-Nieto et al. (2022a), who developed an articulation between the Extended Theory of Connections (ETC) and the Onto-semiotic Approach to Mathematical Knowledge and Instruction (OSA), see Figure 7.

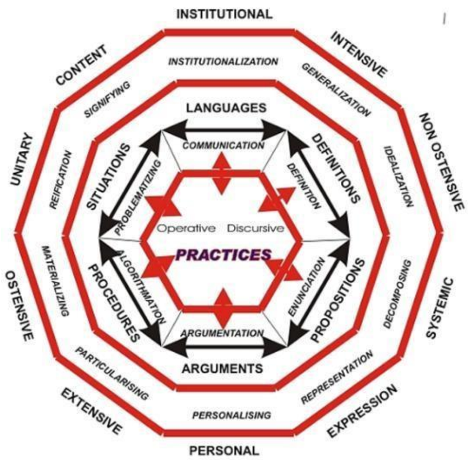


Figure 7. Schematization of mathematical knowledge from an onto-semiotic view (Font & Contretas, 2008).

Next, in Rodríguez-Nieto et al. (2022a), the study focused on three fundamental axes: (i) the analysis of the nature of mathematical connections from the perspective of each theory; (ii) the exploration of these connections through the subjects' written and oral productions, considering operational and discursive dimensions; and (iii) a content analysis of key publications from both frameworks, aimed at identifying relevant principles, methods, and research questions. Likewise, the existence of congruences and complementarities between ETC and OSA was examined with the aim of enriching the understanding of the phenomenon of mathematical connections.

The detailed analysis of these connections was developed following the theoretical integration methodology proposed by Drijvers et al. (2013), Kidron & Bikner-Ahsbahs (2015), and Radford (2008), which consists of the selection and description of episodes that allow for a more precise identification of mathematical connections based on both theories. In the study by Rodríguez-Nieto et al. (2022a), the data were analyzed using OSA tools, such as practices, primary object configurations, and semiotic facets (SF) that relate them. Subsequently, elements of mathematical activity (practices, primary objects, SF) were encapsulated as specific types of connection according to the typology proposed by ETC (e.g., connections of meaning, feature, procedural, part-whole, among others).

It should be noted that, although the analytical methods differ thematic analysis in the case of ETC and analysis of mathematical activity in the case of OSA the results converge on a fundamental conclusion: both theoretical frameworks complement and enhance each other, offering a richer and more articulated view of the phenomenon of mathematical connections. From an OSA perspective, a mathematical connection can be understood as the "tip of the iceberg" of a more complex structure involving practices, processes, primary objects, and the SF that support them. This metaphor makes it possible to visualize that, behind a seemingly simple connection, lies a dense network of interrelated elements that can be revealed through onto-semiotic tools.

The theoretical articulation between ETC and OSA is not restricted to a single mathematical domain but has demonstrated its applicability in diverse contexts: from concepts of Differential Calculus to areas such as Vector Calculus (Rodríguez-Nieto et al., 2024), ethnomathematical connections and geometry (Rodríguez-Nieto et al., 2023a), differential equations (Dans-Moreno et al., 2022), and integrative approaches such as ethnomathematics and the STEAM model (Rodríguez-Nieto & Alsina, 2022). In the present study, the application will focus on the analysis of connections that emerge around concepts such as vectors, partial derivatives, and applications of the chain rule, curl, and divergence.

The articulation between ETC and OSA offers a theoretical framework capable of enriching cognitive analyses from other theories, such as APOS. OSA provides tools to identify the semiotic and epistemic configuration of mathematical constructions involved in the internalization and encapsulation processes described by APOS, while ETC categorizes the connections that mediate these transformations such as meaning, process, or part-whole connections aligned with Dubinsky et al. (2005). Additionally, ETC and OSA expand Duval's Theory of Semiotic Representation Registers (2006) by identifying the type of mathematical connection involved in representation conversion visual, meaningful, or procedural while OSA contributes a deeper understanding of the objects, meanings, and practices in play. This integrated perspective broadens the explanatory power of existing theories by incorporating cognitive, semiotic, and contextual dimensions.

In this sense, the ETC+OSA synthesis acts as a metatheoretical framework that enhances the scope of Duval's theory by inserting the semiotic dimension into a broader conceptual network, capable of revealing not only how mathematical concepts are represented, but also why, for what purpose, and from what epistemic or institutional structure these representations acquire meaning and generate meaningful connections. ETC not only provides an integrated perspective that articulates semiotic, epistemic, and cognitive dimensions of mathematical knowledge, but also constitutes a robust platform for advancing the construction of a broad and grounded typology of mathematical connections, as well as for strengthening other theoretical frameworks based on new readings and conceptual interactions.

A sociocultural view of ETC: Ethnomathematical connections

Ethnomathematical connection refers to the relationship between the mathematical knowledge people use in daily activities and formal mathematics taught in schools (Rodríguez-Nieto, 2021a). These connections are categorized into three types: internal, external, and ethnomathematical meaning (Rodríguez-Nieto, 2020; Rodríguez-Nieto et al., 2023a). Internal connections relate how a person connects different units of measurement within the same system for equivalences and conversions. External connections occur when a measurement unit is used similarly across different systems in daily practices. Ethnomathematical meaning connects a person’s cultural interpretation of a mathematical concept linked to objects or measurements in their context (Rodríguez-Nieto, 2020). These connections support mathematics teaching and learning from various perspectives.

- They are relevant because they value mathematics in the daily practice carried out by a person where the researcher identifies a connection and relates it to institutionalized mathematics.
- They favor the understanding of mathematical concepts considering that the student solves mathematical problems based on real life and, in turn, suggestions on connections from curricular organizations are shared (MEN, 2006).
- They can not only be recognized in a single daily practice, but in several, from the same sociocultural context or from different towns, regions or countries, avoiding the local aspect of ethnomathematics when it is emphasized in a single daily practice (Rodríguez-Nieto & Escobar-Ramírez, 2022, p. 998-999).
- They contribute to the construction of ethnomathematical sequences and the design of tasks (Mansilla et al., 2023), which allows relating basic learning rights to the mathematics identified in daily practices.
- Ethnomathematical connections show that culturally rooted practices activate brain areas linked to language and symbols, like Broca’s and Wernicke’s, even in non-academics. This

highlights how contextualized activities stimulate mathematical reasoning through everyday cognitive and neurocognitive processes.

Ethnomathematical connections can also emerge from the activation of the universal activities proposed by Bishop (1999), but it is important to highlight that each community or person has their local activities with which they carry out their daily practice (Figure 8).

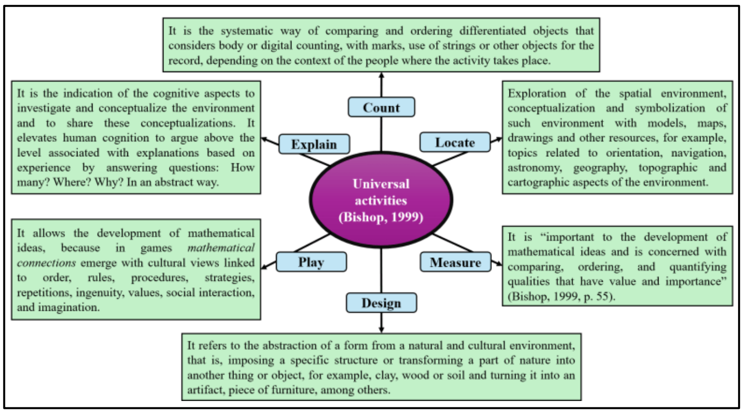


Figure 8. Universal activities used in several daily practices (Adopted from Bishop, 1999).

After theoretically presenting the description of the thematic analysis method, in the first phase, the interviews were organized, transcribed and studied in depth. Also, the field notes and photographs taken by the authors, as well as the written productions, the oral arguments and the gestures of the participants were considered, and the data was triangulated. In the second phase, the codes (C) in the transcripts were identified as shown in Table 1. As an example, only relevant codes extracted from the responses to task 1 will be shown, where words and phrases were identified where the mathematical connections.

Table 1. Excerpts from the transcript of interview with the participants (P) when they solved task 1.

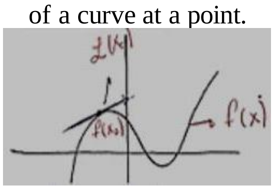
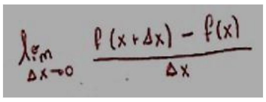
P	Codes	Excerpt from transcription (evidence of codes)
P1	C1	The derivative is the tangent line to a function.
	C2	The derivative is the limit of a function plus an increment minus the original function between the increment.
P2	C3	The derivative is the slope of the tangent line at a point.
	C4	The derivative is the limit, when we have $\Delta x \to 0$ of the limit of the function $f(x)$ plus the increment (Δx), minus the original function between (Δx), that is the definition.
P3	C5	A limit of Δx when it tends to zero of $f(x)$ plus Δx minus $f(x)$ between Δx.
	C6	The derivative represents the slope of the tangent line at a point.
	C7	The derivative is that both the value of y changes with respect to x, that is, an increase or decrease in how much y varies with respect to x and the function evaluated at that point x is the image.
P4	C8	The limit of a quotient that meets certain conditions, when we set a point P_0 on the curve and make another point P that is variable on the curve, then we draw a line that passes through those two points and what comes next is that the P since it can move along the curve then it can move closer to P_0, so as P tends to P_0 a line is formed which is called a tangent, so this tangent makes an angle with the x axis, has a tangent, so this is given that the slope of the line is going to be equal to the tangent of the angle.
	C9	So, the derivative is going to be the slope of the tangent line at a point.
P5	C10	The derivative can be expressed as the slope of the tangent line of a curve at a given point.

C11	The derivative in Calculus is defined as $\lim_{\Delta x \to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x}$
C12	The derivative can be used to measure whether a function is increasing or decreasing.
C13	We use the derivatives of a function to determine the trihedron of a curve. When we have a curve in the third dimension, it is important to associate a reference plane to each point of the curve, and this is made up of, tangent vector, normal vector and binormal vector.

Source: Information taken from Rodríguez-Nieto et al. (2021a)

Subsequently, from Table 2 the codes were grouped into subthemes from third phase. In this process of assigning themes, similarities, characteristics, and meanings associated with the derivative that the participants gave in common were taken into account, which is considered a mathematical connection of meaning and different representations (Table 2, Figures 9 and 10). It is worth noting that the topics were initially reviewed and named by experts in mathematical connections (phase 4).

Table 2. Assignment of subthemes and themes (Rodríguez-Nieto et al., 2021a).

Codes (C)	Subthemes	Theme: Mathematical connection
C1	The derivative is the slope of the tangent line to the curve at a point.	Meaning
C3		
C6		
C9		
C10		
C13		
C2	The derivative is the limit of the of the quotient of the mean rates of variation.	Meaning
C4		
C5		
C8		
C11		
C7	The derivative is the instantaneous rate of change.	Meaning
C10	The derivative can be expressed as the slope of the tangent line of a curve at a point.  **Figure 9**. Connection of different representations (representations: verbal-graphic-symbolic).	Meaning and different representations
C11	The derivative as a limit.  **Figure 10**. Connection of different representations (representations: verbal-graphic-symbolic).	

Source: Information taken from Rodríguez-Nieto et al. (2021a)

It is important to mention that, in the same argument offered by the PSMT (in this case P5) it is possible to identify more than one mathematical connection. For example, by the production of the PSMT we identify in C10 and in C11 the connections of different meaning and representations (see Table 2).

Regarding onto-semiotic analysis, a case is shown with the same participating student in the work of Rodríguez-Nieto (2021b) and Rodríguez-Nieto et al. (2022a), highlighting the power of establishing links between two types of analysis that help to identify mathematical connections in a more detailed way in terms of mathematical practices, processes, objects and semiotic functions that relate them (see Figure 11).

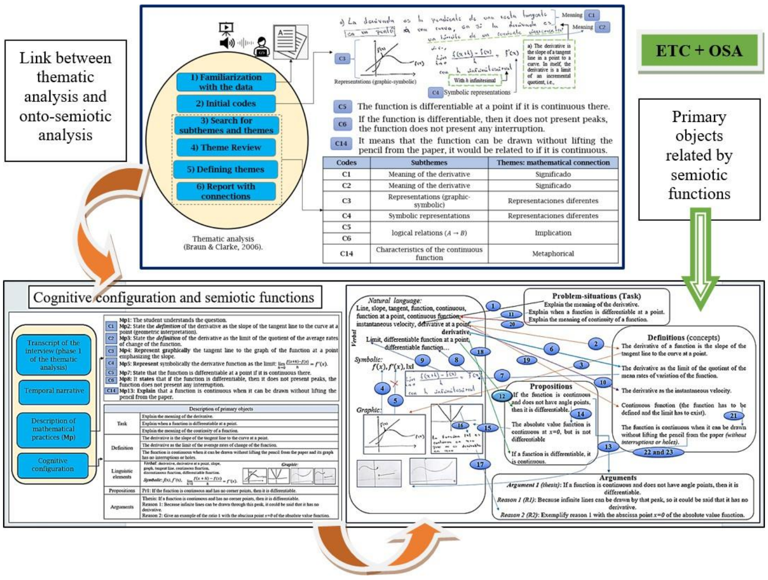


Figure 11. Synthesis of the ontosemiotic analysis of mathematical connections (Taken from Rodríguez-Nieto et al., 2022c).

A particular case of connections is the meaning of the derivative at a point, such as the slope of the line tangent to the curve at that point, and is summarized in Figure 12. The subject can value the connection as the tip of an iceberg made up of a conglomerate of practices, processes, objects and semiotic functions.

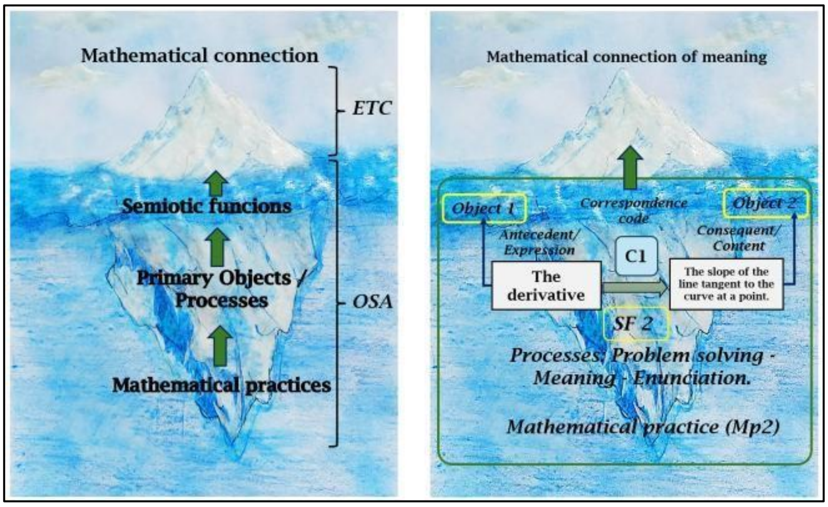


Figure 12. Synthesis of the connection of meaning.

On the other hand, in the ETC there are extensions linked to the appreciation of mathematics practiced by cultural groups (ethnomathematics) (D'Ambrosio, 2001), but this theory delves deeper into the connection of sociocultural aspects with institutional mathematics present in textbooks and other curricular materials. Figure 13 presents an example of ethnomathematical connections, given that in different everyday practices (coffee, bean, and corn plantings), the unit of measurement is the sack or sack, equivalent to 100 kilograms.

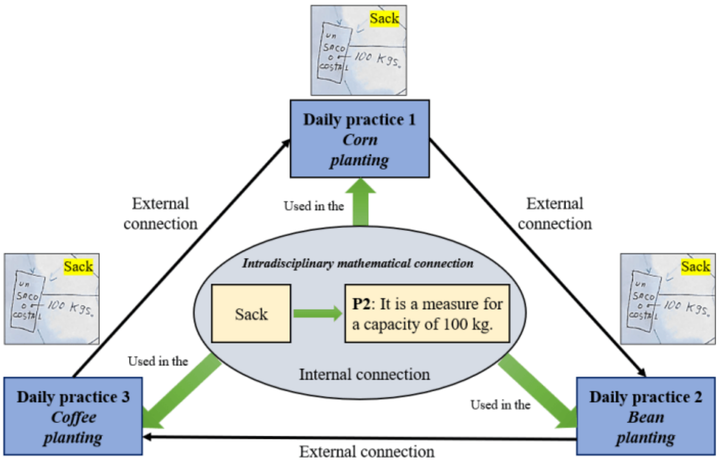


Figure 13. Example of ethnomathematical connections (Rodríguez-Nieto & Alsina, 2022)

Other ethnomathematical connections have been identified in the use of artisanal stoves in the preparation of *sancocho de guandú* (soup) from Sibarco, Atlántico, Colombia, connecting these artifacts to the preparation of gastronomic products and 2D and 3D geometry (see Figure 14).

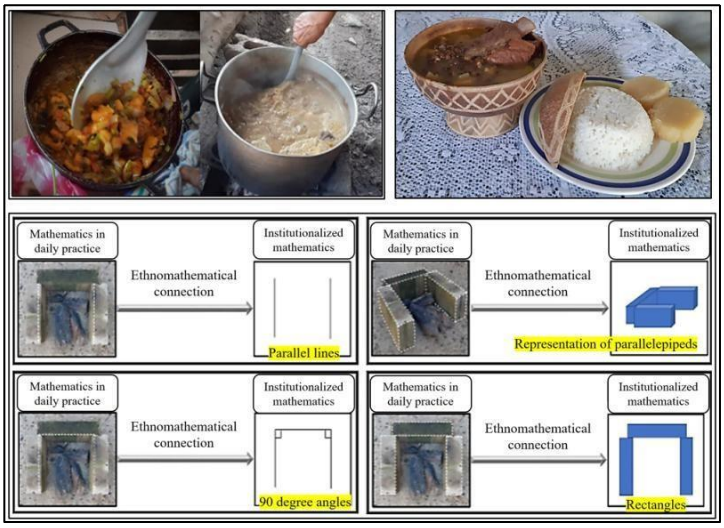


Figure 14. Ethnomathematical connections between stoves and geometric figures (Taken from Rodríguez-Nieto & Escobar-Ramírez, 2022).

Finally, this theory has been further extended with the assessment of the cognitive aspects of connections, which is why a networking between the ETC combined with the OSA and neo-mathematics has been carried out to give rise to neuro-mathematical connections (Cantillo-Rudas, 2025; Cantillo-Rudas & Rodríguez-Nieto, 2024; Cantillo-Rudas et al., 2024). For example, neuro-mathematical connections are recognized in the mathematical activity of students and teachers when they solve a geometric problem, highlighting the first connection as made in Cantillo-Rudas et al. (2024) with the neuro-mathematical connection of term and symbol recognition (Figure 15). This type of connection is activated when the student reads the statement, visual areas in the occipital and temporal lobes are activated, aiding in symbol and word recognition. Simultaneously, the brain regions involved in language comprehension (Wernicke's area) and production (Broca's area) are engaged. These areas, connected by the arcuate fasciculus, process word meanings and their relationships. This activation enables the student to attribute meanings and characteristics to the object such as interpreting a box as a parallelepiped based on language and visual processing (Cantillo-Rudas et al., 2024).

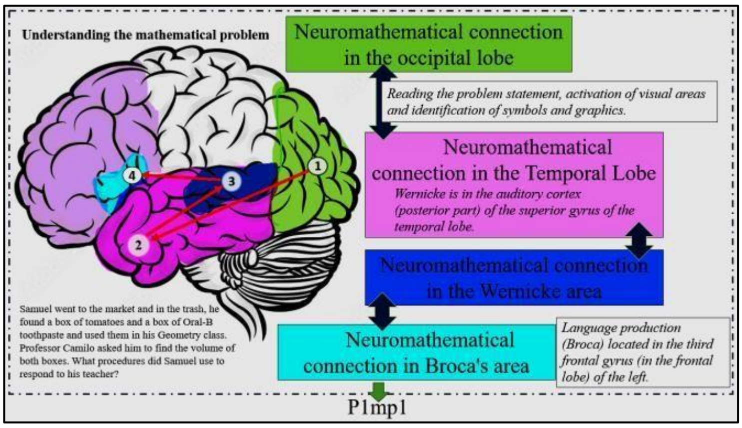


Figure 15. Neuro-mathematical connections for P1mp1 (taken from Cantillo-Rudas et al., 2024).

Discussion

This article synthesizes key contributions of the Extended Theory of Connections (ETC) to Mathematics Education, in dialogue with the Onto-semiotic Approach (OSA). The iceberg image illustrates how mathematical connections are constructed at various depths, from visible practices to abstract cognitive processes mediated by semiotic functions. ETC tools enable the analysis of both explicit and emerging connections across expressions, meanings, and contexts, including mathematical, neuro-mathematical, and ethnomathematical connections. This integrative approach addresses the disconnection between school mathematics and meaningful everyday situations, offering a powerful framework to rethink mathematics teaching and learning through a connective perspective.

One of the main contributions of ETC is its dynamic and expansive nature. As new semiotic functions and mathematical objects emerge in student and teacher activity, the theory can be enriched by incorporating new connection categories, enabling more refined analyses. This flexibility has earned ETC recognition in national and international literature, appearing in high-impact journals (Q1 and Q2), confirming its relevance as a contemporary framework for analyzing mathematical knowledge in action. Its articulation with OSA deepens the understanding of meaning-making in connections. For example, the image shows a meaning connection between "the derivative at a point" and "the slope of the tangent line," linked by a correspondence code activated in problem-solving. This interpretation aligns with the onto-semiotic analysis of mathematical objects and processes, showing that every meaningful connection is mediated by practices, meanings, and representations.

Without extending this reflection further, it is essential to emphasize that the development of ETC represents an unfinished collective endeavor. Despite the advances achieved in combination with OSA, there is a need to continue refining its theoretical and methodological tools through new research efforts. Some of these challenges have already been addressed through ethnomathematics (Rodríguez-Nieto & Alsina, 2022; Sudirman et al., 2024), ethnomodelling (Rodríguez-Nieto et al., 2022), cycle of modeling (Ledezma et al., 2024), neuromathematics (Cantillo-Rudas, 2025), mathematical argumentation and proof (Rodríguez-Nieto et al., 2023c), and meaningful learning (Rodríguez-Nieto et al., 2025a), among others.

Conclusions

Mathematical connections are vital in both education and daily life, as they help individuals interpret and solve real-world problems using mathematical knowledge (Berry & Nyman, 2003). These connections enhance logical, critical, and creative thinking by linking abstract concepts to everyday experiences such as managing personal finances, interpreting media graphs, or understanding scientific phenomena. They also support informed decision-making in various contexts (Rodríguez-Nieto, 2021a). In cultural practices, people integrate empirical knowledge with institutionalized concepts, showing that connections are more than teaching tools they are means to understand the world mathematically (Rodríguez-Nieto et al., 2025a). International curricula increasingly recognize this perspective, promoting mathematical connections as a key transversal competence for problem-solving and knowledge integration (NCTM, 2000; MEN, 2006; Campo-Meneses & García-García, 2023). Therefore, encouraging the development of mathematical connections from early to higher education is essential for cultivating critical, reflective citizens capable of navigating everyday challenges with a solid mathematical foundation.

Ultimately, the ETC especially in its articulation with the OSA constitutes a robust conceptual platform for analyzing mathematical connections across multiple dimensions: cognitive, semiotic, cultural, and neurocognitive. Its strength lies in enabling analyses that go beyond superficial content exploration to delve into the deep structures of mathematical meaning in action. In this way, it stands as a key tool for building a more connected, inclusive, and meaningful Mathematics Education capable of responding to the epistemological, pedagogical, and social challenges of the 21st century.

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