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Zone of Concept Image Differences in the Concept of Angles Formed by A Transversal at Undergraduate Level

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ABSTRACT

This study explored the differences in undergraduate students' concept images related to angles formed by a transversal intersecting any two lines compared with concept definition. Using diagnostic tests and interviews, the qualitative study with phenomenological design examined various student representations and common error patterns. Students' answers were analyzed qualitatively to identify patterns, misconceptions, and variations in their concept images, followed by semi-structured interviews to explore their justifications. Participants were 35 second-semester students from the mathematics education study program at a state university in Aceh, Indonesia, who had completed the plane geometry course. The findings revealed significant variations in students' concept images, which were: (1) pairs of alternate interior/exterior angles, corresponding angles, and same-side interior/exterior angles were formed when two parallel lines were intersected by a transversal; (2) the measure of corresponding angles must be equal; (3) the measure of alternate interior angles must be equal; (4) the measure of alternate exterior angles must be equal; (5) the measure of same-side interior angles must be equal; and (6) misunderstanding and incorrectly identifying alternate exterior angles. These results highlight the importance of adapting teaching approaches to address differences in concept images and to better support students in mastering geometric concepts. The novelty lies in its use of the Zone of Concept Image Differences to analyze the gap between students' concept images and formal definitions, offering insights into how to bridge these gaps in teaching.

Keywords: angles formed by a transversal, concept definition, concept image, geometry, zone of concept image differences.

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Introduction

The relationships between pairs of angles formed when two lines intersected by a transversal are among the fundamental concepts in geometry. Pairs of angles are alternate interior angles, alternate exterior angles, same-side interior angles, same-side exterior angles, and corresponding angles (Alexander & Koeberlein, 2020; Clemens et al., 1990; Usiskin et al., 2003). It also looks at the measures of these angle pairs when the transversal intersects two parallel lines. From this exploration, several other important topics in geometry can be derived, such as the theorem on the sum of interior angles in a triangle, theorems on the properties of quadrilaterals, and the concept of similarity (Moise, 1990; Prenowitz & Jordan, 1965). This implies that a proper understanding of the concepts of angles formed by a transversal is crucial for mastering these subsequent concepts. Therefore, it can also be stated that teaching the concept of angles formed by two lines intersected by a transversal is highly significant. This instruction must be carried out systematically. Otherwise, it may hinder students' processes of constructing knowledge. Such obstacles are referred to as didactical obstacles (Brousseau, 1997; Suryadi, 2019).

The previous research indicated that many issues persist regarding students' mastery of geometry concepts. In a meta-analysis article, Budiarto & Artiono (2019) identified several problems related to geometry, including misconceptions among students in learning geometry. Other studies have revealed that prospective teachers have limited knowledge of geometry (Aslan-tutak & Adams, 2015; Marchis, 2012). In particular, with reference to the idea of angle pairs formed when a transversal intersects two lines, Baidoo & Baidoo (2022) discovered that preservice mathematics teacher students frequently lack grasp of corresponding and alternate angles, demonstrate inaccurate reasoning related to parallelism and transversals, have limited knowledge for justifying problems about parallelism, and hold misconceptions about parallel lines. These findings suggest that students exhibit diverse conceptual understandings of corresponding and alternate angles and display inadequate knowledge of parallelism. Students' perception of corresponding and alternate angles may differ since their concept representations are frequently in conflict with the formal concept definitions.

In mathematics education, The term "concept image" refers to a student's mental representation of a mathematical concept, which contains all of the visual, symbolic, and sensory qualities associated with that notion (Tall & Vinner, 1981; Vinner, 1991, 2020). This varies from the concept definition, which is the formal, explicit statement of a mathematical idea, sometimes offered in textbooks or by teachers. The gap between students' idea representations and formal definitions is a well-documented issue in learning mathematics, leading to misunderstandings and misconceptions (Duval, 1995). Tall & Vinner (1981) emphasize that concept images can include "all the mental pictures and associated properties" that students form over time through problem-solving, visual representations, and classroom discussions. This mental representation may not always align with the formal definition provided in textbooks or taught by teachers. Duval (1995) further elaborates that many students struggle to translate their intuitive visual images into precise formal reasoning, especially in geometry, where spatial and symbolic reasoning are intertwined.

The exploration of concept images and their differences from formal definitions is essential for improving students' understanding of mathematical concepts. A concept image encompasses all mental representations, experiences, and associations related to a mathematical concept (Tall & Vinner, 1981; Vinner, 1991). These images, formed through prior learning experiences and informal understandings, often deviate from the precise, formal definitions presented in the curriculum (Edwards & Ward, 2004). This divergence can lead to persistent misconceptions, hindering students' ability to apply concepts accurately.

Several previous studies have explored students' and pre-service teachers' concept images across various topics. Sulastri et al. (2021) examined the concept of limits and found that students understood the limit of a function as an unreachable value. This misconception led to an inadequate understanding of the formal definition of limits using ε-δ. Similarly, Siagian et al. (2021), in their research on variables, discovered that most students perceived a variable as an unknown value, while some viewed it as a substitute for an unknown value. In another study on the topic of derivatives, it was found that students' concept images of derivatives were limited to the representation of functions. Regarding the meaning of the derivative, most participants viewed it merely as a tool for solving procedural problems (Prihandhika et al., 2022). Another study also explored the undergraduate students' concept images in calculus course and how to construct the valid concept images (Ojo & Olanipekun, 2023).

In the specific context of angles formed by a transversal, previous studies have highlighted that students frequently misidentify relationships such as alternate interior or corresponding angles, often relying on incorrect or incomplete mental models (e.g., Duval, 1995; Presmeg, 1986). For instance, Duval (1995) emphasized the role of visualization in geometry, noting that students' reliance on visual cues rather than formal reasoning could exacerbate misunderstandings of geometric relationships. Moreover, Hershkowitz (1990) noted that geometric concepts like angles and transversals were abstract and required significant cognitive processing to relate visual representations to formal definitions. Unlike earlier research, this study focuses on the investigation of concept images and the conceptual gap of angles created by a transversal. By adopting a phenomenological approach, this study seeks to uncover students perceive, interpret, and construct their understanding, shedding light on potential learning obstacles that may arise in the learning process (Mertens, 2020).

These challenges underscore the importance of investigating students' concept images, particularly in areas where visual and formal reasoning intersect, such as angles and transversals. By identifying the specific zones of differences between concept images and formal definitions, teachers or educators can design targeted interventions to address these gaps, as suggested by Sfard (1991) in her work on mathematical concept development. Understanding students' Concept Images can also suggest improvements in teaching that address the formation of incorrect Concept Images, making the learning process more effective and meaningful (Vinner, 1983).

Based on the background above, it is evident that students' concept images often deviate from formal definitions, posing significant obstacles to their comprehension of geometric concepts. To examine in detail the differences between concept definitions and concept images, it is necessary to analyze the Zone of Concept Image Differences (ZCID). ZCID draws attention to the disparity or differences between the formal concept definitions and the conceptual images that students have. This analysis helps identify areas where students' understanding deviates from scientifically accepted definitions and offers insights into how these gaps can be addressed in teaching (Siagian et al., 2021; Sulastri et al., 2022). ZCID analysis can guide the creation of more effective learning experiences that bridge the gap between students' incorrect mental representations and the accurate mathematical definitions (Prihandhika et al., 2022; Vinner, 1991). To address this issue, this study aimed to explore and analyze these discrepancies. Consequently, the research question was: how did students' concept images of angles formed by a transversal differ from the formal concept definitions?

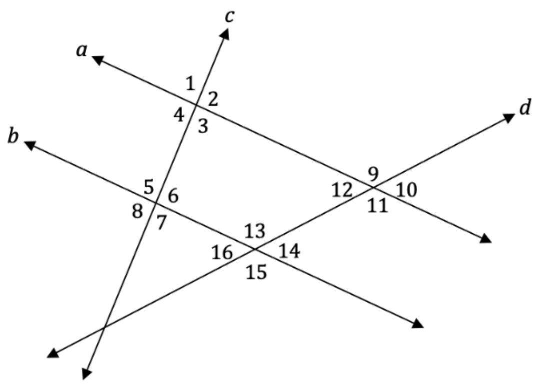
The novelty of this study lies in its focus on examining the Zone of Concept Image Differences (ZCID), a framework for identifying and analyzing the gap between students' conceptual representations and formal geometric definitions. This method provides a thorough knowledge of how students' mental representations of geometric ideas, such as angles formed by a transversal, differ from scientifically established definitions. By investigating these inconsistencies, the study provides new insights into how such gaps might be handled in education, with the goal of improving the alignment between students' informal conceptions and formal mathematical notions.

Methods

In this study, a qualitative study was conducted, employing a phenomenological research design. The fundamental goal of this study was to explain a thorough and detailed description of the phenomena associated with Zone of Concept Images Differences (ZCID) or the gap between students' conceptual understanding and formal geometric concepts. Qualitative research was used because it was regarded a helpful tool for providing a complete and in-depth description of a certain program, activity, or place (Mertens, 2010). Furthermore, phenomenological research was selected as the research design method since it was an approach where the researcher sought to document and explain people's actual experiences in connection to a certain event, as stated by the participants themselves (Giorgi, 2009; Moustakas, 1994). This method enables a thorough investigation of how people interpret and understand a specific occurrence, offering insightful information about their viewpoints and experiences.

Participants were selected using purposive sampling techniques, namely a technique for selecting participants who are considered to be able to provide information that is in accordance with the phenomena or symptoms to be studied or observed (Creswell, 2014; Fraenkel et al., 2012; Mertens, 2010). Thirty-five second-semester students from a state university in Aceh's mathematics education study program participated in this study. They were selected as participants because they had completed the plane geometry course, ensuring they had prior exposure to the concept of angles and transversals. To explore students' concept images of angles formed when two lines are cut by a transversal, a test with three questions was given to the students (see Figure 1).

Given that $a \parallel b$ and $c \nparallel d$ as shown in the picture below.



1. Determine the name of the following pair of angles (if any).
   - $\angle 1$ and $\angle 5$
   - $\angle 3$ and $\angle 15$
   - $\angle 7$ and $\angle 16$
   - $\angle 1$ and $\angle 11$
2. If any, determine one pair of corresponding angles and alternate exterior angles for lines a and b intersected by transversal c.
3. If any, determine one pair of alternate interior angles and same-side interior angles for lines c and d intersected by transversal a.

***Figure 1**. Test Questions*

Using praxeology, part of the Anthropological Theory of Didactic (ATD) (Chevallard & Bosch, 2020), all of the tasks above could be organized into praxis block, i.e., task (T), techniques (τ), and logos block, i.e. technology (θ), and theory (Θ) (See Table 1). Type of task (T) indicates a given problem or situation, while technique (τ) states how to solve a given problem or how to work on steps from a given situation. Technology (θ) refers to the reasons for the technique used, and theory (Θ) is used to justify or explain the technology (technique used) (Chevallard, 2005). The use of this theory ensured that the test questions were well-justified and directed toward the theory or concept whose Zone of Image Concept Difference (ZCID) was being examined.

Table 1. *Praxeological Organization of the Test Questions*

Task ($\mathbf{T}$)	Techniques ($\boldsymbol{\tau}$)	Technology ($\boldsymbol{\theta}$)	Theory ($\boldsymbol{\Theta}$)
$\mathbf{T_1}$: Determining name of angles formed by a transversal (in condition parallel and non-parallel)	τ_1: Observe the position of the angle pairs in question, identify the lines and the transversal that form these angle pairs, and then determine to which type of angle pair they belong.	θ_1: Definition of alternate interior angles, alternate exterior angles, corresponding angles, and same-side interior angles, and same-side exterior angles	Θ: Corresponding angles: Two non-adjacent angles that are on the same side of a transversal and are designated as interior and exterior, respectively
$\mathbf{T_2}$: Determining the corresponding angle pair and alternating exterior angles when two parallel lines cross each other transversally	$\tau_{2.1}$: Determine which angles are on the same side of the transversal but are not contiguous by looking at the lines and the transversal in question. One angle is an interior angle, while the other is an exterior angle. $\tau_{2.2}$: Observe the lines and the transversal in question, and then Find two external angles that are not contiguous but are on the other side of the transversal.	$\theta_{2.1}$: Definition of corresponding angles $\theta_{2.2}$: Definition of alternate exterior angles	Alternate interior angles: two internal angles on opposite sides of a transversal that are not contiguous. Alternate exterior angles: two external angles on opposing sides of a transversal that are not contiguous Same-side interior angles: two interior angles located on the same side of the transversal
$\mathbf{T_3}$: Determining the alternate interior and same-side angles pair interior when two parallel lines cross each other transversally	$\tau_{3.1}$: Observe the lines and the transversal in question, and then Find two internal angles that are not contiguous but are on the other side of the transversal. $\tau_{3.2}$: Observe the lines and the transversal, and then find the two interior angles that are located on the same transversal side.	$\theta_{3.1}$: Definition of alternate interior angles $\theta_{3.2}$: Definition of same-side interior angles	Same-side exterior angles: two external angles that are located on the same side of the transversal (Alexander & Koeberlein, 2020; Clemens et al., 1990; Lewis, 1968; Moise, 1990)

The students' written responses were qualitatively analyzed to identify patterns and misconceptions. Based on this, five students were selected for semi-structured interviews to explore their reasoning in more depth. The research procedure is summarized in Figure 2.

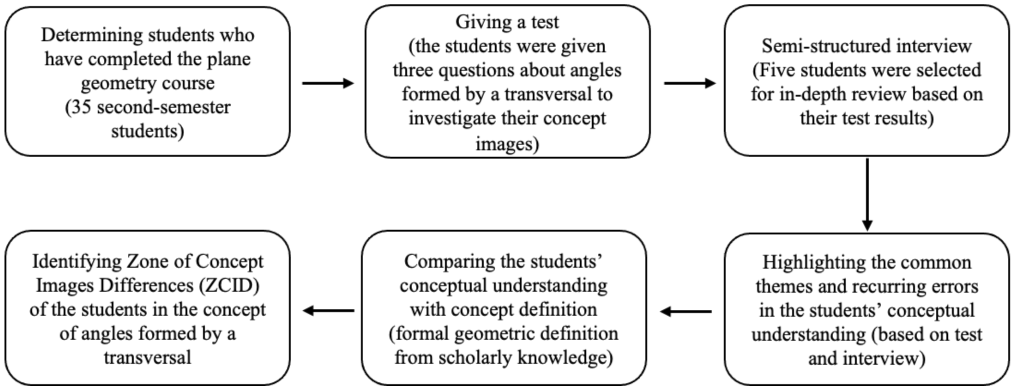


***Figure 2**. Research Procedure*

Results and Discussion

Overview of the students' answers

Task 1

In task 1, students identified names of angle pairs formed by various line configurations—some with parallel lines, others without. The goal was to assess their ability to correctly name angle pairs and reveal misconceptions, such as assuming certain angle properties only apply with parallel lines. For the pair of angles $\angle 1$ and $\angle 5$, the students were able to identify the type of angle pair as corresponding angles. During follow-up interviews, all students agreed that these were corresponding angles because two parallel lines, lines a and b, were intersected by a transversal, line c. For the pair of angles $\angle 3$ and $\angle 15$, there were at least two types of responses from the students: (1) there was no name for the angle pair because they are on non-parallel lines, and (2) they were alternate exterior angles. For the first answer, the student correctly stated that there was no special name for the pair of angles $\angle 3$ and $\angle 15$. But their reasoning was incorrect in saying that the absence of a name for the pair of angles was because they were not parallel lines cut by a transversal. However, when considering the various angle pairs formed when two parallel lines are intersected by a transversal, $\angle 3$ and $\angle 15$ do not have a specific relationship or a designated name as a pair. The second unique response, "alternate exterior angles," was explored further through interviews to uncover the reasoning behind it. The interviews revealed that the students labelled the angle pair $\angle 3$ and $\angle 15$ based on their observation of the diagram. From the figure given in the question, it appeared that $\angle 15$ was positioned opposite $\angle 3$ and located outside the lines, leading to the name "alternate exterior angles." Figure 3 showed the illustration of the students' explanation.

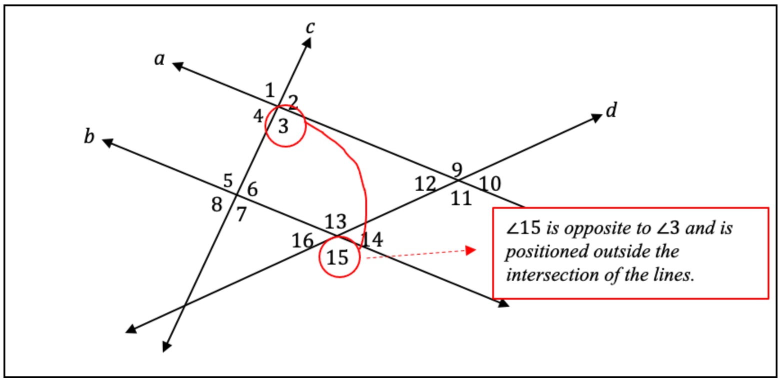
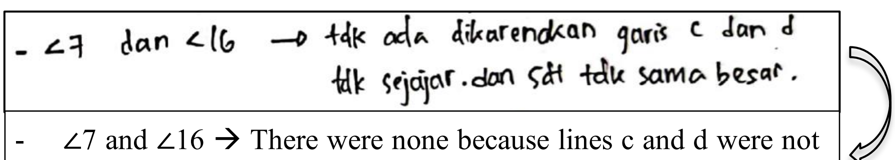


Figure 3. *An illustration of the students' explanation regarding the angle pair ∠3 and ∠15*

For the angle pairs ∠7 and ∠16 as well as ∠1 and ∠11, the students' responses were relatively similar: they stated that there was no name for these angle pairs because the lines were not parallel. However, when analyzed, ∠7 and ∠16 are actually same-side interior angles for lines *c* and *d* with transversal *b*. Similarly, ∠1 and ∠11 were alternate exterior angles for lines *c* and *d* with transversal *a*. Students remained fixated on the condition of parallelism as a prerequisite for forming same-side interior angles and alternate exterior angles. In reality, when discussing angle pairs like ∠7 and ∠16 or ∠1 and ∠11, it is important to understand that the formation of these angle pairs does not always depend on the lines being parallel. Angle pairs can be determined based on the properties of the transversal intersecting two lines, whether the lines are parallel or not. From the Figure 4, it can also be observed that, in addition to the lines not being parallel, some students believed that angles ∠7 and ∠16 did not have a specific name because their measures were not equal. There is a conceptual error here, as students incorrectly assumed that angle congruence was a requirement for forming same-side interior angle pairs.



- ∠7 and ∠16 → There were none because lines c and d were not parallel and the angles were not congruent

Figure 4. Students' answers about the relationship ∠7 and ∠16

Task 2

In Task 2, the students were asked to determine one pair of corresponding angles and one pair of alternate exterior angles when lines a and b (parallel to each other) were intersected by transversal c. The goal of this task was to examine students' comprehension of the concepts of corresponding and alternate external angles. Many students were able to provide accurate answers. One example of their answers can be seen in Figure 5. The results showed that students generally understood corresponding and alternate exterior angles when parallel lines are cut by a transversal. Interviews confirmed that they linked these angles to the parallelism of lines a and b. However, some relied only on visual cues, recalling textbook or classroom diagrams without fully grasping the underlying definitions and geometric principles.

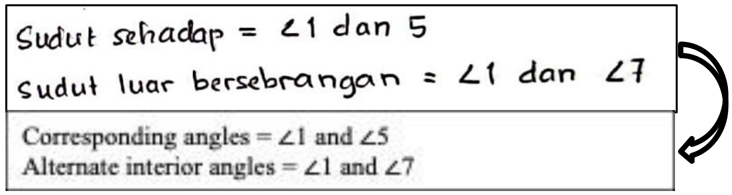


Figure 5. *Example of students' correct answers*

Task 3

Similar to Task 2, students were required to identify a certain set of angles in Task 3—in this case, same-side interior angles and alternate interior angles. However, the fundamental difference compared to Task 2 lied in the condition of the lines: lines c and d were not parallel. By working through this task, students must examine the formation of these angles in a non-parallel context. Some students answered that there were no angle pairs because lines c and d were not parallel.

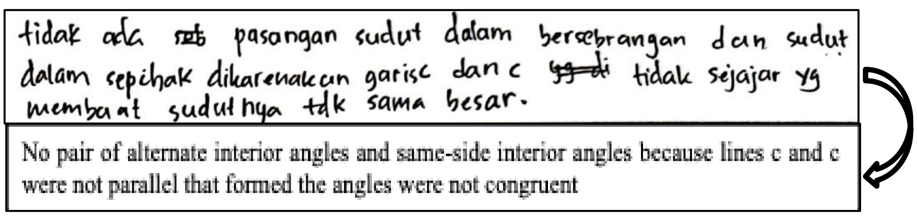


Figure 6*. Students' answer about the need for parallel lines to form certain pairs of angles*

In addition, from the answers in Figure 6, it can be seen that students assumed that the congruence of angle pairs, such as alternate interior angles, was a requirement for the existence of those angles. If the angles were not congruent, the angle pairs were considered non-existent. This is corroborated by the interview results, where students firmly believed that this was correct—that alternate interior angle pairs and other angle pairs, such as corresponding angles, must always have congruent measures. Another problem found from the students' answers above was the absence of same-side interior angles regarding non-congruent angles.

Another common mistake in students' answers was incorrectly identifying the lines and the transversal. This occurred when students misinterpreted the positions of the lines c and d in relation to the transversal a, leading to incorrect conclusions about the angle pairs. For the example, as shown in the Figure 7, ∠3 and ∠5 was pair of alternate interior angle and ∠4 and ∠5 was pair of same-side interior angle. The answer was correct for the lines a and b intersected by transversal *c*. But the question was for lines *c* and *d* intersected by transversal *a*.

Figure 7. *Example of students' incorrect answer caused by error in identifying the lines and the transversal*

Analysis of Zone of Concept Image Differences (ZCID)

From the explanation above, several concept images have been formed. These concepts differ from the scholarly concepts commonly found in geometry reference books. In this section, the results of the Zone of Concept Image Differences (ZCID) analysis will be explained.

Concept Image 1: Pairs of alternate interior/exterior angles, corresponding angles, and same-side interior/exterior angles were formed when two parallel lines were intersected by a transversal

The main gap between students' concept image and the formal definition lies in the conditions for forming angle pairs. Students often believe such pairs only occur with parallel lines, whereas formally, these angles arise whenever a transversal intersects any two lines, parallel or not (Alexander & Koeberlein, 2020; Leonard et al., 2014; Lewis, 1968; Moise, 1990). The area where concept definition and concept image diverge is hence parallelism. Students' reliance on familiar diagrams, where lines are almost always parallel, can lead to false associations. They often assume angle pairs like corresponding or alternate interior angles only exist with parallel lines, due to overemphasis on special cases in teaching. This reflects an epistemological obstacle rooted in how the concept was first introduced without sufficient contextual variation (Brousseau, 1997).

Concept Image 2: The measure of corresponding angles must be equal; if they were not equal, the angles could not be considered as a pair of corresponding angles

Students often assume corresponding angles are always equal, whereas formally, they are equal only when formed by a transversal intersecting parallel lines (Leonard et al., 2014; Lewis, 1968; Moise, 1990). Students often misbelieve that corresponding angles are always equal, overlooking that congruence occurs only when lines are parallel. This misconception arises from instruction that introduces angle pairs solely in parallel contexts. Such limited exposure reflects a didactical obstacle, where learning difficulties stem from how content is sequenced and presented (Brousseau, 1997).

Concept Image 3: The measure of alternate interior angles must be equal; if they were not equal, the angles could not be considered as a pair of alternate interior angles

Students often believe that alternate interior angles must be congruent to exist, assuming their validity depends on congruence. Formally, however, such angles exist whenever a transversal intersects two lines, regardless of angle measures (Lewis, 1968). Congruence occurs only when these angles are formed by parallel lines or vice versa (Clemens et al., 1990; Wallace & West, 1998). This zone shows a common misconception: students believe alternate interior angles must be equal to exist. This zone reveals a common misconception: students think alternate interior angles must be equal. In fact, such angles exist whenever a transversal cuts two lines—their equality depends on parallelism, not existence. This misunderstanding stems from limited learning focused only on parallel lines, creating an epistemological obstacle (Brousseau, 1997).

Concept Image 4: The measure of alternate exterior angles must be equal; if they were not equal, the angles could not be considered as a pair of alternate exterior angles

Alternate exterior angles form on opposite sides of a transversal intersecting two lines. Their congruence occurs only when the lines are parallel (Clemens et al., 1990; Lewis, 1968), but the existence of these angle pairs is not depend on their congruence. Students often assume alternate exterior angles must be congruent to exist, rejecting unequal pairs. This misconception stems from early visual introductions using parallel lines, not formal definitions. As a result, they believe congruence defines existence. While visualization aids learning (Arcavi, 2003), overreliance may hinder conceptual understanding. Visualization that is not accompanied by proper explanation has the potential to cause misunderstanding. DePiper & Driscoll (2018) stated that the mathematics teachers' use of visualization representation is complex and requires hard abilities such as a solid understanding of the material, anticipating students' thinking, and selecting the optimal visual representation to utilize with students. Educators must understand the consequences of using specific representations and when they are appropriate (Ball et al., 2008).

Concept Image 5: The measure of same-side interior angles must be equal; if they were not equal, the angles could not be considered as a pair of same-side interior angles

Students mistakenly believe that same-side interior angles are always equal when two lines are cut by a transversal. Figure 8 highlights this misconception, where students assume any such angles must have equal measures. In fact, same-side interior angles are located between two lines on the same side of a transversal, and their measures vary unless the lines are parallel. In parallel lines, these angles are supplementary (Alexander & Koeberlein, 2020; Clemens et al., 1990; Leonard et al., 2014), and only congruent if the transversal is perpendicular. This error often arises from students generalizing patterns seen in other congruent angle pairs, leading to flawed concept images (Vinner, 2020).

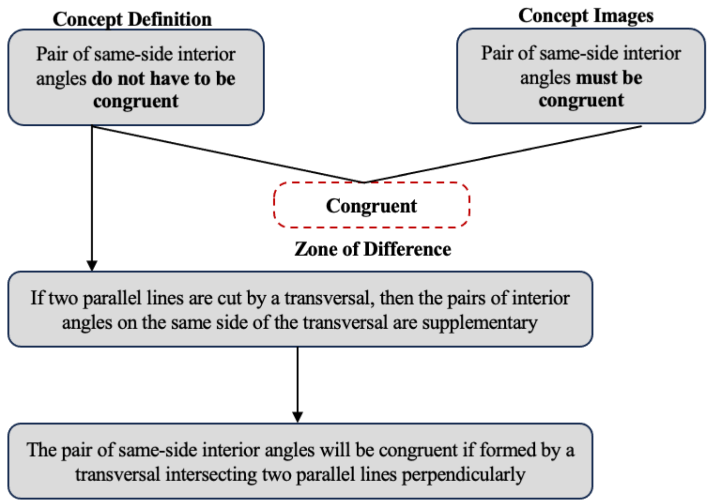


Figure 8. *Illustration of zone of difference for concept image 5*

Concept Image 6: Alternate exterior angles were a pair of angles where one angle was opposite to another and was positioned outside the intersection of the lines. It can involve two transversals

Students mistakenly believe alternate exterior angles involve two transversals and are simply opposite angles outside the lines. Figure 9 and Figure10 show this misconception, which deviates from the formal definition. This confusion likely stems from misinterpreting terms like "opposite" and "alternate," highlighting the importance of precise vocabulary for mathematical understanding (Riccomini et al., 2015; Seethaler et al., 2012).

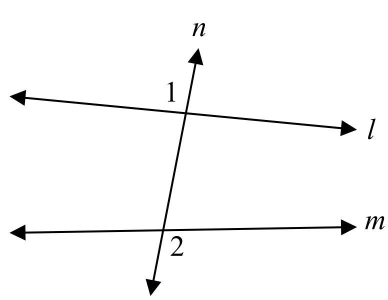


Figure 9. *Illustration of alternate exterior angles based on concept definition*

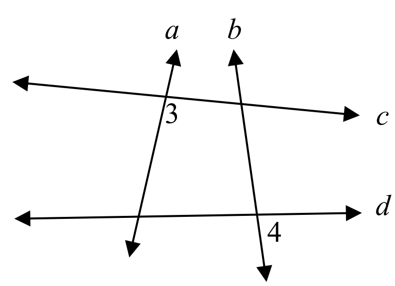


Figure 10. *Illustration of alternate exterior angles based on students' concept image*

The Zone of Concept Image Differences (ZCID) reflects the gap between students' concept image and the formal definition. In didactic transposition theory (Chevallard, 1989), this gap contrasts learned knowledge with scholarly knowledge. The process of didactic transposition may lead to changes in meaning or simplifications of a concept (Chevallard, 1989, 2019). If many students develop a different understanding from the intended concept, this may indicate an issue in the didactic transposition process. A good didactic transposition procedure will effect on giving the relevant mathematical concepts and generating an acceptable learning environment (Jamilah et al., 2020). ZCID results from ineffective didactic transposition that fails to align with students' understanding. The presence of ZCID suggests that students' understanding is influenced not only by formal instruction but also by cognitive structures, and the representations provided during the learning process. These differences can lead to misconceptions, difficulties in problem-solving, and a lack of coherence in students' mathematical reasoning. This must be considered during the learning process, since Abdullah et al. (2015) and Herizal et al. (2019) a lack of knowledge of the concepts can impair students' abilities to solve mathematical problems.

Conclusion

This study underscores the critical gap between students' concept images and formal concept definitions in the context of angles formed by a transversal intersecting two lines. The findings reveal several misconceptions, such as misidentifying angle pairs, misinterpreting the criteria for corresponding and alternate angles, and incorrectly classifying alternate exterior angles, which are caused by a variety of factors such as visual misinterpretations, rule overgeneralization, and a lack of exposure to diverse geometric scenarios during the learning process. These concerns indicate that students frequently rely on inadequate or erroneous mental representations rather than formal definitions, resulting in repeated errors and a lack of conceptual clarity.

The differences in students' conceptual images highlight the critical need for targeted instructional strategies that explicitly address these gaps, such as incorporating dynamic visualization tools, engaging students in reasoning and justification tasks, and providing diverse examples and non-examples. Adopting such approaches allows educators to ensure that students gain precise, adaptable, and thorough understandings of geometric concepts, resulting in more successful and engaging geometry learning experiences. Future study is required to establish a didactical design to avoid the gap between students' concept images and concept definitions. This study has several limitations that need to be considered. First, this study focuses on a specific group of students, so the findings obtained may not be generalizable to a wider population. In addition, this study mainly examines the differences between concept image and concept definition in the context of angles formed by a transversal, without exploring other geometric relationships in depth. In terms of methodology, this study uses interviews and written tests as the main instruments, which can cause subjectivity in data interpretation because they do not include direct observation or experimental approaches.

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