

Proportional Reasoning of Junior High School Students in Solving Geometric Similarity Problems Based on Mathematical Disposition

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ABSTRACT

Proportional reasoning is the foundation of mathematical abilities, including algebra, geometry, and statistics, and is influenced by students' mathematical dispositions. The transition from additive to multiplicative reasoning plays a crucial role in problem-solving. This study explores junior high school students' strategies in understanding geometric similarity through a qualitative case study involving three students in Surabaya selected through purposive sampling based on high, medium, and low mathematical dispositions. This study uses a qualitative approach with a case study method. The subjects of the study consisted of three eighth-grade students purposively selected to represent high, medium, and low mathematical disposition, allowing an in-depth examination of how different dispositions influence proportional reasoning. The instruments used were similarity problem worksheets and semi-structured interview guidelines. Data were collected through student answers and interviews, then analysed through data reduction, data presentation, and drawing conclusions. The results showed differences in proportional reasoning strategies according to mathematical disposition: S1 at level 3 (Formal Reasoning), S2 at level 2 (Quantitative Reasoning), and S3 at level 0 (Non-Proportional Reasoning). Proportional reasoning develops when quantity coordination and multiplicative strategies are used in an integrated manner, in line with each student's mathematical disposition. Student with high disposition consistently use ratio-based multiplicative strategies, student with medium disposition use multiplicative strategies, while student with low disposition tend to use additive rules or random approaches. These findings are exploratory and can serve as a basis for studying misconceptions and developing proportional learning.

Keywords: mathematical disposition, proportional reasoning, similarity

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Introduction

Proportional reasoning is a key indicator of mathematical proficiency and a core concept in K-12 education, as it underpins learning across algebra, geometry, and statistics (Ergene & Karaboğaz, 2024; Sari et al., 2023). Its development is closely linked to students' overall mathematical growth, particularly through the transition from additive to multiplicative thinking, which requires understanding numbers in relational rather than isolated ways (Proulx, 2023). Although additive reasoning forms an essential foundation, insufficient mastery can hinder the development of number sense and limit students' ability to engage effectively in proportional problem solving (Kumm & Graven, 2024). Accordingly, additive and multiplicative reasoning are interrelated components that support students' proportional competence,

with targeted interventions shown to enhance multiplicative thinking and adaptive problem-solving skills (Määttä et al., 2022; Shaver & DeJarnette, 2024).

Conceptually, proportional reasoning involves analyzing multiplicative relationships among ratios, fractions, and rates, identifying consistent patterns between quantities, and distinguishing proportional from non-proportional situations (Açikgöl, 2021; Gündoğdu & Tunç, 2022). Students employ a range of strategies, from procedural approaches such as cross multiplication to more flexible methods like unit rates and pattern recognition, which reflect different developmental levels of proportional reasoning. To determine the extent of each student's proportional reasoning ability, this ability can be classified into four levels, from level 0 to level 3 (Langrall & Swafford, 2000; Sari et al., 2023). However, difficulties persist when instruction emphasizes procedures without sufficient conceptual grounding (Aktaş, 2022; Fernández et al., 2024). Proportional reasoning also serves as a foundation for algebraic thinking and is closely connected to geometric similarity, where constant ratios between corresponding sides define similar figures (Arican & Özçakir, 2021; Burgos et al., 2024; Öztürk et al., 2021).

Beyond cognitive factors, students' proportional reasoning is influenced by mathematical disposition, which includes affective and behavioral tendencies such as perseverance, engagement, and responses to challenge. This disposition affects how students select strategies, persist in resolving cognitive conflict, and flexibly interpret relationships between quantities (Ibrahim et al., 2023). Proportional reasoning itself can be categorized into ratios and rates, proportional relationships, and non-proportional relationships, understanding of which is essential to avoid linearity assumptions and promote flexible thinking (Begolli et al., 2021; Gea et al., 2023; Hernández-Solís et al., 2023; Martínez-Juste et al., 2023). Understanding these categories is important for promoting flexibility in students' thinking when solving various types of mathematical problems.

Although proportional reasoning has been widely researched, no study has specifically addressed junior high school students' proportional reasoning strategies in the context of geometric similarity based on mathematical dispositions. Previous studies focused on prospective teachers (Arican & Özçakir, 2021), social arithmetic problems without a geometric context (Izzatin et al., 2021), teachers' interpretation of proportional situations (Nugraha et al., 2023), proportional reasoning as a predictor of probability learning (Begolli et al., 2021), and quantitative studies, textbook analysis, and mathematical literacy without examining students' thinking strategies in similarity and their relationship to mathematical disposition (Linuhung et al., 2024; Son et al., 2022; Supply et al., 2023). Therefore, this study analyzes junior high school students' proportional reasoning strategies in geometric similarity problems based on mathematical dispositions to fill the existing research gap.

Methods

This study uses a qualitative approach with a case study method to explore the proportional reasoning used by students in solving similarity problems (Creswell & Poth, 2018). Case studies were chosen because they allow for in-depth conceptual organization of the phenomenon being studied with a focus on specific contexts and cases, and support the formulation of research questions that evolve from the researcher's perspective to the students' perspective (Stake, 2010). This allows researchers to examine the process of proportional reasoning with an emphasis on a deep understanding of the phenomenon, rather than on statistical generalizations.

This study involved three eighth-grade students from a junior high school in Surabaya. In case study-based qualitative research, the number of participants is not determined by considerations of statistical representativeness, but rather by the need to gain an in-depth understanding of each case studied. The case study approach is holistic and oriented towards the case as a limited system, so this study does not aim to make comparisons between subjects, but rather to intensively examine the proportional reasoning process that arises in each subject in its natural context (Stake, 2010).

Subject selection was conducted using purposive sampling with a maximum variation approach. Three students were selected based on data from teachers, indicating that they had different mathematical dispositions: high, medium, and low. The first student had a high disposition and was expected to demonstrate reflective reasoning and consistency in coordinating changes in quantity. The second student had a medium disposition and tended to apply procedural reasoning, with efforts to maintain consistency in the relationships between quantities. The third student had a low disposition, typically using an additive or partial approach, so that limitations in maintaining proportional relationships were clearly evident.

This approach is not intended to make comparisons between subjects, but rather to reveal patterns of proportional reasoning that emerge intensively and contextually in each case, so that consistent patterns and meaningful differences can provide a richer picture of the core experiences and shared dimensions of students' proportional reasoning in solving similarity problems (Patton, 2010).

Mathematical disposition in this study is understood as students' positive attitudes and behaviors toward mathematics, including curiosity, self-confidence, perseverance, flexibility, and enjoyment in solving mathematical problems (Kilpatrick et al., 2001). Students were grouped into categories of high, medium, and low mathematical disposition qualitatively based on an analysis of their behavior in completing similarity tasks and interview results. The indicators used included confidence, perseverance, willingness to try alternative strategies, and reflection on the answers obtained. The interpretation of these

disposition categories was validated through discussions with mathematics teachers using the member checking technique to increase the credibility of the analysis results (Patton, 2010).

This study has limitations in terms of the representativeness and generalization of the findings, given the limited number of subjects and the specific research context. However, in accordance with the qualitative paradigm, which aims to understand the meaning of individuals or groups assigned to social or human problems through emergent procedures, inductive analysis, and interpretive inquiry (Creswell, 2009).

This study uses levels of proportional reasoning as a conceptual framework to explore students' strategies and approaches in solving similarity problems. These levels provide categories that can be used to map students' thinking from the simplest to the most complex (Langrall & Swafford, 2000; Sari et al., 2023). **Table 1** shows the indicators of proportional reasoning levels used as a reference.

Table 1. *Indicators of Proportional Reasoning Levels*

Level	Indicator
0 (Non-Proportional Reasoning)	Unable to recognize or use proportional relationships, and the strategies used are additive or random.
1 (Informal Reasoning)	Unable to recognize or use proportional relationships, and the strategies used are additive, guesswork, or dependent on props/pictures.
2 (Quantitative Reasoning)	Able to use proportional relationships with the help of models or concrete tools, and the strategies used begin to be multiplicative.
3 (Formal Reasoning)	Able to construct and solve proportions symbolically using variables, cross-multiplication rules, or equivalent fractions, and the strategies used are formal in nature, understanding the structural relationships between invariant and covariant quantities.

In **Table 2**, questions are designed with real-life contexts to engage students in proportional reasoning. Each item is tailored based on the classification of proportional reasoning (Martínez-Juste et al., 2023).

Table 2. *Instrument Items Designed*

Proportional Reasoning Type	Indicator	Instrument Item
Proportional Relationship	Used a magnification constant to determine a new size that was directly proportional to the original size.	Item number a
Ratio and Rate	Maintained the ratio between two quantities so that the shape remained similar after a change in size.	Item number b
Non-proportional Relationship	Linked changes in size squared to changes in resource requirements	Item number c

Similarity Problem

A painter created a 20 cm \times 40 cm painting of a flower. The painting was so popular that he wanted to make a large print version of the painting while maintaining similarity so as not to distort the proportions of the image. The large print is planned to have an area 4 times that of the original painting.

- Determine the length and width of the large mold so that they remain similar!
- Explain how you determined these measurements and why your calculations maintain the similar.
- If when making the initial flower painting a total of 50 ml of paint is needed, how much paint is needed to make a large print painting?

After the students completed the problem, a semi-structured interview was conducted to explore their proportional reasoning process. The students interviewed were selected based on the variety of answers that showed different levels of proportional reasoning. The data analysis flow is shown in **Figure 1**.

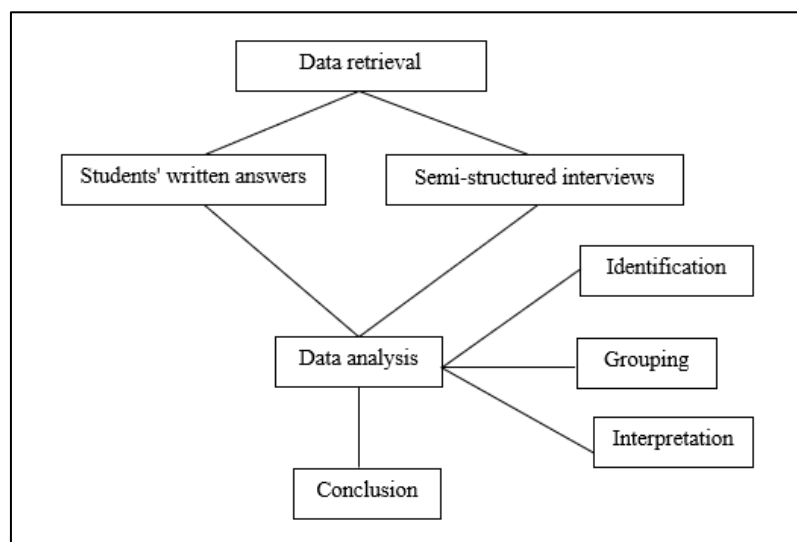


Figure 1. Data Analysis Flow

Results and Discussion

Based on the steps that have been taken in this study, including data collection, analysis, and interpretation of the subjects' answers, the results obtained can be presented as follows.

a) Student with High Mathematical Disposition (S1)

Subject S1's use of proportional reasoning in solving the similarity problem is illustrated in the responses and interview excerpts shown in **Figure 2**.

Jawaban:

a.

b. karena rasio dari Luasan dan cetakan itu 1:2

c. $50 \times 4 = 200$ ml
karena diperbesar

$$\begin{aligned} \text{Luas lukisan} &= 20 \times 40 = 800 \\ \text{Luas cetakan} &= 3200 \\ &= (k \times 20) \times (k \times 40) = 3200 \\ k^2 \times 800 &= 3200 \\ k^2 &= 4 \\ k &= 2 \end{aligned}$$

$$(2 \times 20) \times (2 \times 40) = 40 \times 80$$

$$L = 40 \text{ cm}$$

$$P = 80 \text{ cm}$$

Translate:

Answer:

a. Painting area = $20 \times 40 = 800$

Area of the mold = 3200

$$(k \times 20) \times (k \times 40) = 3200$$

$$k^2 \times 800 = 3200$$

$$k^2 = 4$$

$$k = 2$$

$$(2 \times 20) \times (2 \times 40) = 40 \times 80$$

Explanation:

Width = 40 cm

Length = 80 cm

b. Because the ratio of painting and print is 1 : 2

c. $50 \times 4 = 200$ ml

Because it is enlarged

Figure 2. Results of Subject S2's Work

R : "When you first read the problem, what came to mind?"

S1 : "Back in elementary school, the problems were usually asked to find the size of the new shape after enlarging or reducing it and often had to find the ratio of length or area."

R : "Have you ever encountered a problem (more than 1) similar to this? If so, what was the gist of the problem?"

S1 : "I have, but usually, the questions ask me to find the size of the new shape after it has been enlarged or reduced, and often have to find the ratio of length or area."

R : "In the beginning, did you imagine some ways to solve it? If so, describe those ways!"

S1 : "Yes, one way is to look at the area first and then find the ratio. Another way is to directly use the side length ratio."

R : "Explain, what are your steps in doing the problem?"

S1 : "In point a, the first step is to find the ratio by multiplying the area of the first building by 4, then using the variable k. The value of $k = 2$. Then the size of the shape becomes $40 \text{ cm} \times 80 \text{ cm}$ because the ratio is 1 : 2. The answer to point b also uses this measurement. While point c is done by using equal comparison, which is simply multiplied by 4."

R : "When you are faced with a problem that involves changing the size of a shape, such as enlarging or reducing an object, how do you start the steps of solving it?"

S1 : "The first step is usually to look at the initial size and the size in question, and then figure out the ratio."

R : "In problems that involve changes in size, how do you ensure that the changes are proportional? What do you notice first?"

S1 : "The first thing to look at is whether all sides of the shape change by the same ratio. If yes, then the change is permanent."

R : "Are you sure that your answer is correct? Why?"

S1 : "Sure, because the steps were appropriate and the calculation results matched the comparison used in the problem."

Based on **Figure 2**, S1 shows a structured numerical thought pattern by starting with an area comparison. S1 calculates the area of the original painting as $20 \times 40 = 800$, and then compares it to the area of the print, which is 3200. From this comparison, S1 determined that the area change factor was 4. To maintain proportionality, S1 modeled the size change using the variable k by writing $(k \times 20) \times (k \times$

40) = 3200. This form is $k^2 \times 800 = 3200$, $k^2 = 4$ and $k = 2$. This scale factor is then applied to each side, resulting in a new size of 40 cm \times 80 cm. The same pattern is also used for other quantities, namely the amount of paint, by multiplying 50 ml by the area change factor. So that 200 ml is obtained.

From the interview, it can be seen that S1 consistently pays attention to the initial size and the question, ensuring that all sides change by the same ratio so that the changes are proportional, and is confident in the answer because the steps are appropriate and the calculation results are consistent. This pattern shows that S1 associates changes in area, changes in sides, and changes in other quantities through a consistent ratio. S1's proportional reasoning is at level 3 (formal reasoning) because they not only apply calculation procedures but also maintain proportional relationships between quantities and verify the results. A high mathematical disposition supports the integration of conceptual mastery and creative thinking skills. These findings are in line with previous research emphasizing the importance of quantity coordination and multiplicative strategies as ratio relations (Izzatin et al., 2021; Lamberg et al., 2021; Sari et al., 2023)

b) Student with Medium Mathematical Disposition (S2)

Subject S2's use of proportional reasoning in solving the similarity problem is illustrated in the responses and interview excerpts shown in **Figure 3**.

<p>Jawaban:</p> <p>A. $LB = 20 \text{ cm} \times 90 \text{ cm}$ $= 9 \times \text{lipat}$ $= 20 \times 9$ $= 80 \text{ cm}$ $= 90 \times 9$ $= 160 \text{ cm}$ $= 80 \text{ cm} \times 160 \text{ cm}$ jadi cetakan besar memerlukan $80 \text{ cm} \times 160 \text{ cm}$</p> <p>B. Dengan lukisan bunga awal dikali 9, karena rencana memiliki luas 9x lipat, lukisan harus sejajar, kalau senisai tidak lukisan jelek.</p> <p>C. JK = 50 ml cat $LB = (20 \text{ cm} \times 90 \text{ cm}) \times 9$ $= 80 \text{ cm} \times 160 \text{ cm}$ $= 50 \times 9$ $H = 200 \text{ ml cat}$ jadi cat yang dibutuhkan 200 ml cat</p>	<p>Translate: Answer:</p> <p>a. Large Painting = 20 cm \times 40 cm Fourfold • $20 \times 4 = 80 \text{ cm}$ • $40 \times 4 = 160 \text{ cm}$ So, a large mold requires 80 cm \times 160 cm. The student multiplied each side by 4, so the size became 80 cm \times 160 cm</p> <p>b. The initial flower painting is multiplied by 4 because the comparison has an area of fourfold. The painting must be aligned, if it is not aligned, it is bad.</p> <p>c. If: 50 ml of paint Large painting = $(20 \text{ cm} \times 40 \text{ cm}) \times 4$ $= 80 \text{ cm} \times 160 \text{ cm}$ Resultant amount of paint = $50 \times 4 = 200 \text{ ml}$ of paint</p>
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Figure 3. Results of Subject S2's Work

- R : "When you first read the problem, what came to mind?"
 S2 : "When I first read the question, I immediately thought it was easy because I had seen questions like this before."
 R : "Have you ever encountered a problem (more than 1) similar to this? If so, what was the gist of the problem?"
 S2 : "Once, when I was in elementary school. The problem was similar, discussing two flat shapes."

- R : “At the beginning, did you imagine some ways to solve it? If so, describe those ways!”
 S2 : “Only one way comes to mind, the same one that has already been done.”
 R : “Explain, what are your steps in solving the problem?”
 S2 : “For point a, multiply each by 4, so the size is $80\text{ cm} \times 160\text{ cm}$. For point b, it is taken from the result of the multiplication, and for point c, it is also multiplied by 4.”
 R : “So, you're not paying attention to whether the area matters?”
 S2 : “No, so just multiply it.”
 R : “In problems that involve changes in size, how do you ensure that the changes are proportional? What do you notice first?”
 S2 : “Multiplied by the same number.”
 R : “Are you sure that your answer is correct? Why?”
 S2 : “Not all sure, still a little hesitant, especially in question A. Because I didn't pay attention to the area and make sure the ratio was the same.”

Based on **Figure 3**, S2 began the solution by focusing on the initial size of the painting, which was $20\text{ cm} \times 40\text{ cm}$, then immediately used a multiplier of 4 for each side to obtain a new size of $80\text{ cm} \times 160\text{ cm}$. The pattern of thinking that emerged was to multiply by 4 all quantities involved, including length, width, and amount of paint. S2 did not first review the explicit relationship between areas, but relied on the similarity of the multiplier as the main reason that the size change was correct. This was also reflected in the paint calculation, where $50\text{ ml} \times 4 = 200\text{ ml}$. From the interview, it appears that S2 chose this strategy because they had encountered a similar problem before and assumed that a proportional change in size could be achieved simply by multiplying all quantities by the same number.

Although consistent in using multiplication, S2 has not fully coordinated the relationship between quantities reflectively or symbolically. S2's proportional reasoning is at level 2 (quantitative reasoning). Medium mathematical creative disposition is related to S2's proportional reasoning pattern, which is characterized by the unstable integration of creative thinking and creative disposition, as well as the tendency of student to respond to tasks when the context or structure of the problem is clear, but not yet independently exploring. These findings are in line with research emphasizing that multiplicative strategies are meaningful when ratio relationships are understood, while inconsistencies in quantity coordination indicate a medium level of mathematical creative disposition (Izzatin et al., 2021; Linuhung et al., 2025)

c) Student with Low Mathematical Disposition (S3)

Subject S3's use of proportional reasoning in solving the similarity problem is illustrated in the responses and interview excerpts shown in **Figure 4**.

Jawaban: a. $40 \text{ cm} \times 60 \text{ cm}$
 b. Menurut saya dgn jumlah segitu ($40 \times 60 \text{ cm}$) itu adalah porsi yg pas untuk mempertahankan kesebangunan
 c. 200 ml

Translate:
 Answer:
 a. $40 \text{ cm} \times 60 \text{ cm}$
 b. In my opinion, this amount ($40 \text{ cm} \times 60 \text{ cm}$) is the right portion to maintain the alignment.
 c. 200 ml of paint

Figure 4. Results of Subject S3's Work

- R : "When you first read the problem, what came to mind?"
 S3 : "Two shapes that are the same."
 R : "Have you ever encountered a problem (more than 1) similar to this? If so, what is the gist of the problem?"
 S3 : "I have but I have forgotten. I remember in elementary school."
 R : "In the beginning, did you imagine some ways to solve it? If so, describe those ways!"
 S3 : "Only what is done like this."
 R : "Explain, what are your steps in solving the problem?"
 S2 : "For point a, the length and width are different by 20, so I looked for another size with a difference of 20, so the size is $40 \text{ cm} \times 60 \text{ cm}$. Point b is also the same because the difference is fixed, then point c is just multiplied by 4."
 R : "When you are faced with a problem that involves changing the size of a shape, such as enlarging or reducing an object, how do you start the steps of solving it?"
 S3 : "Still confused."
 R : "In problems that involve changes in size, how do you ensure that the changes are proportional? What do you notice first?"
 S3 : "Don't understand yet."
 R : "Are you sure that your answer is correct? Why?"
 S3 : "No, but sure about point c."

Based on **Figure 4**, S3 gave the answer of $40 \text{ cm} \times 60 \text{ cm}$ and 200 ml of paint. The numerical thinking pattern used by S3 is based on a fixed difference between length and width, namely 20 cm. S3 states that because the difference between length and width in the initial size is 20, other sizes that are considered appropriate must also have the same difference, resulting in $40 \text{ cm} \times 60 \text{ cm}$. For the amount of paint, S3 directly multiplies by 4 without explaining the basis for the quantitative relationship, other than following the previous step. From the interview, it appears that S3 uses a strategy intuitively based on previous experience, without clear strategic planning, showing hesitation when asked to explain the initial steps or how to ensure proportional changes.

S3's strategy shows an additive and non-proportional approach, because the multiplication does not represent a multiplicative relationship between quantities, but rather a separate procedure without justification. S3's proportional reasoning is at level 0 (non-proportional reasoning). These findings are consistent with previous research showing that non-proportional student tend to maintain a fixed difference and use procedural multiplication without maintaining the ratio (Nugraha et al., 2023).

Overall, the three subjects showed different strategies that reflected their respective mathematical dispositions. S1 was more reflective and consistently maintained the ratio between quantities; S2 was procedural and not yet reflective, while S3 used an additive and intuitive approach without maintaining the ratio. These findings suggest that proportional reasoning develops when a student is able to consistently coordinate two quantities, rather than simply applying multiplication operations. The multiplicative strategy becomes meaningful when understood as a ratio relationship, while the additive approach emerges when the relationship between quantities is not maintained (Izzatin et al., 2021; Lamberg et al., 2021; Nugraha et al., 2023).

Furthermore, previous research emphasizes that the ability to coordinate quantities, integrate multiplicative strategies, and reflective mathematical dispositions support the development of students' proportional reasoning (Burgos et al., 2024; Deal et al., 2025; Ibrahim et al., 2023). These findings indicate that proportional reasoning develops when students are able to consistently coordinate changes in two quantities, rather than simply applying multiplication operations. This pattern shows that multiplicative strategies become meaningful when understood as ratio relationships, rather than as separate procedures. Conversely, when the relationship between quantities is not maintained, the approach used tends to be additive even though it involves multiplication operations. These findings are in line with previous studies emphasizing that the higher the students' mathematical disposition, the higher the level of proportional reasoning they demonstrate (Izzatin et al., 2021).

However, this study has a major limitation in terms of the number of subjects, as each mathematical disposition category is represented by only one student. Therefore, the findings are not intended to represent internal variations within each category or to be generalized to a broader student population. Therefore, the results of this study are positioned as an exploratory qualitative study that emphasizes a deep understanding of the individual proportional reasoning process in a specific context.

Conclusion

The use of proportional reasoning in this study demonstrates its role in helping three research students with different mathematical dispositions (high, medium, and low) solve geometric similarity

problems. The findings of this case study reveal that student with high mathematical dispositions tend to use multiplicative reasoning strategies and consistent side-to-side comparison relationships, resulting in systematic and accurate problem solving. Student with medium mathematical dispositions showed the use of proportional strategies that were not yet fully stable, with multiplicative reasoning. Meanwhile, student with low mathematical dispositions tended to use random strategies or simple additive rules without considering scale relationships, which led to errors in problem solving.

Given that this study used a case study design with a limited number of subjects, the findings are exploratory in nature. However, the differences in proportional reasoning patterns identified at each level of mathematical disposition provide an important picture of the variation in student strategies in the context of geometric similarity. The implications of these findings indicate the need to develop geometric similarity learning that explicitly emphasizes the transition from addition reasoning to multiplication reasoning by considering differences in students' mathematical dispositions.

Based on these findings, further research is recommended to examine more specifically the forms of misconceptions in proportional reasoning that arise at each level of mathematical disposition, particularly the tendency to use additive reasoning in problems that require multiplicative reasoning. In addition, further research can focus on developing learning approaches that support students' transition from additive reasoning to multiplicative reasoning by considering differences in mathematical disposition as an important factor in the learning process.

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