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Commognitive Conflict: How do Critical Thinkers Solve Cognitive Conflict Problems in Geometry?

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ABSTRACT

There has been no research exploring cognitive conflict problems in geometry from commognitive framework. Nevertheless, this framework offers strong potential for gaining new theory about cognitive processes of critical thinkers. This study addresses this gap by exploring in depth how critical thinkers solve cognitive conflict problems in geometry from commognitive framework. Commognitive involves four main components: word use, visual mediators, routines, and narratives. This study employed a qualitative approach to explore the cognitive processes in depth. The instrument used in this study consisted of cognitive conflict problems in geometry designed for junior high school students. The subjects of this study consisted of 17 students from the mathematics olympiad group at Madrasah Tsanawiyah (MTs) Surya Buana Malang, Indonesia. The results revealed two categories: Category A met all critical thinking components and commognitive indicators, whereas Category B met only some. The commognitive conflicts highlighted key moments of cognitive engagement and discourse transformation. These conflicts activated critical thinking components, including interpretation, analysis, evaluation, inference, explanation, and self-regulation. Based on the result, it is recommended that future research explore the development of mathematics instructional designs in cognitive conflict problems based on commognitive framework.

Keywords: cognitive conflict, cognitive conflict problems, commognitive, critical thinking, geometry

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Introduction

The reasoning is central to school mathematics learning. According to Kollosche (2021) mathematical reasoning requires consistency, coherence, assessment, argument justification, and proof. Nevertheless, many researchers (Altarawneh & Marei, 2021; Dos Santos, 2019; Dunleavy et al., 2021; Harbour et al., 2022; Hikamah et al., 2021; Ikun et al., 2023; Leikin & Sriraman, 2022; McAvaney, 2022) have emphasized that mathematics learning in schools should have a didactic effect on students' ability to face challenges, acquire advanced knowledge, and enhance their thinking productivity. The structures of mathematical problems should significantly contribute to students' understanding of mathematical content (Colling et al., 2022; Lahdenperä et al., 2022). Furthermore, conceptual understanding plays a crucial role in guiding students' mathematical reasoning and fostering their critical thinking skills.

According to Facione et al. (1994), critical thinking is inherently goal-oriented and results in interpretation, analysis, evaluation, and inference. It involves evidentiary, conceptual, methodological, categorical, and contextual considerations. Critical thinking encompasses a process of analysis and evaluation guided by reasoning rules, ultimately leading to the development of new, deep, and insightful knowledge. Facione identifies six core components of critical thinking: interpretation, analysis, evaluation, inference, explanation, and self-regulation. Interpretation involves developing a holistic understanding of a problem and can be carried out by identifying relevant elements and illustrating them through visual representations such as graphs, diagrams, or concrete objects. Analysis refers to synthesizing the relationships among known elements to generate appropriate arguments and ideas. Evaluation is the process of verifying the validity of reasoning used to support problem-solving. Inference involves gathering data and information to reach logical conclusions. Explanation refers to articulating one's reasoning in a coherent and contextually appropriate manner. Lastly, self-regulation entails an individual's awareness and ability to monitor and control their reasoning processes.

According to researchers, critical thinking is a systematic, active, and purposeful mental process used to analyze, evaluate, and synthesize information to form rational judgments or decisions (Boran & Karakuş, 2022; Dolapcioglu & Doğanay, 2022; Guerra, 2024; Khusna et al., 2024; Rhodes, 2020; Sternberg, 2019). This process involves skills such as identifying problems, clarifying assumptions, assessing evidence, recognizing biases, and considering multiple perspectives before drawing conclusions. According to Khusna et al. (2024) and Boran & Karakuş (2022) individuals with strong critical thinking abilities approach issues from diverse viewpoints, evaluate situations based on sound arguments and verifiable scientific evidence, and actively manage their thought processes by organizing ideas to enable deep exploration and analysis. This competence encompasses a set of cognitive skills, including interpretation, analysis, evaluation, inference, explanation, and self-regulation, which form the foundation for sound decision-making.

In mathematics learning, instructional activities are designed not only to transmit knowledge but also to cultivate students' critical thinking skills. Critical thinking plays a crucial role in fostering creativity, renewal, and innovation in mathematics learning (Álvarez-Huerta et al., 2022; Dolapcioglu & Doğanay, 2022; Umam & Susandi, 2022). Several researchers (Evered, 2020; Movshovitz-Hadar & Hadass, 1991; Wijeratne & Zazkis, 2021) argued that one instructional approach that has been widely recognized as effective in promoting critical thinking is the use of cognitive conflict problems in mathematics learning. Cognitive conflict problems are designed to confront students' existing conceptions with conflicting information, thereby inducing cognitive dissonance. This cognitive dissonance prompts students to interpret the problem, analyze information, evaluate alternative ideas, draw inferences across related topics, regulate and justify their reasoning, and construct new explanations, all of which are core components of critical thinking. Cognitive conflict problems in mathematics promote critical thinking skills by engaging students in analyzing contradictions, evaluating alternative ideas, and constructing deeper mathematical reasoning (Erlich & Gindi, 2019; Kauppi & Drerup, 2021).

Geometry is a fundamental branch of mathematics that plays a crucial role in developing students' mathematical reasoning. However, in real classroom practice at the junior high school levels, geometry is consistently reported as one of the most challenging topics for students (Cesaria & Herman, 2019; Murni et al., 2025; Sudirman et al., 2024). Many students experience difficulties in interpreting geometric representations, coordinating visual and symbolic information, and transitioning from procedural manipulation to conceptual reasoning. These difficulties often manifest as common misconceptions, fragmented understanding of geometric concepts, and reliance on memorized procedures without meaningful justification.

From a commognitive perspective, geometry is particularly prone to generating cognitive conflict because students frequently encounter discrepancies between their intuitive visual perceptions and formal mathematical definitions or properties. Unlike other mathematical topics that rely heavily on symbolic manipulation, geometry requires students to integrate visual mediators, spatial reasoning, and verbal explanations simultaneously (Wijayanto et al., 2024). This integration often exposes conflicts between students' everyday visual reasoning and formal geometric discourse, making geometry an ideal domain for examining commognitive processes. Furthermore, the strong reliance on visual representations, diagrams, and spatial relationships in geometry provides rich opportunities to analyze students' word use, visual mediators, and routines—key components of the commognitive framework. These characteristics position geometry as an appropriate context for investigating how cognitive conflict can foster critical thinking in mathematics learning.

In advancing the understanding of students' reasoning in cognitive conflict situations, Sfard's theory of commognition offers a valuable analytical framework. The term commognition is a fusion of "communication" and "cognition", it is proposes that learning mathematics is a process of discourse transformation (Sfard, 2007). This perspective shifts the focus of learning from individual mental constructs to socially mediated communication, positioning thinking itself as a form of communication. According to Sfard (2007), mathematical thinking can be analyzed as discourse, which evolves in response to the learner's need to express and refine mathematical ideas. The commognitive serves as a tool to capture and interpret these evolving discourses, enabling researchers to investigate students' cognitive processes in depth. Analogous to a microscope that magnifies hidden structures, the commognitive framework reveals subtle but critical elements of mathematical reasoning often overlooked in conventional cognitive approaches. The commognitive involves components in analyzing mathematical discourse: word use, visual mediators, narratives, and routines (Jeannotte & Kieran, 2017; Moustapha-Corrêa et al., 2021; Nachlieli & Tabach, 2022; Pratiwi et al., 2022; Remillard, 2014).

The application of the commognitive is particularly relevant in situations where students solving cognitive conflict problems. These episodes of disequilibrium provide fertile ground for examining how students negotiate meaning, adjust their reasoning, and shift their mathematical discourse in response to conceptual challenges. By employing the commognitive framework, researchers can closely observe these discursive shifts, offering insights into the cognitive processes underlying students' reasoning and problem-solving behaviors.

There has been no research examining cognitive conflict problems in geometry from commognitive framework. Nevertheless, this framework offers strong potential for gaining new theory about cognitive processes of critical thinkers. This study addresses this gap by exploring in depth how critical thinkers solve cognitive conflict problems in geometry from commognitive framework. The urgency of this study lies in its potential to generate new theory about cognitive processes of critical thinkers. The research question is formulated as follows: How do critical thinkers solve cognitive conflict problems in geometry from commognitive framework?

Methods

This study employed a qualitative approach to explore in depth students' cognitive processes when solving cognitive conflict problems in geometry. Commognitive was used to deep analysis of students' cognitive processes. Commognitive involves four main components: word use, visual mediators, routines, and narratives. The indicators of Commognitive was explained in Table 1. The instrument used in this study consisted of cognitive conflict problems in geometry designed for junior high school students. The instrument is presented in Figure 1.

Table 1. *Indicators of Commognitive*

No	Commognitive	Indicators
1	Word uses	Students use mathematical vocabulary and terminology when solving problems.
2	Visual mediators	Students use mathematical symbols, diagrams, figures, and graphical tools to represent and communicate ideas.
3	Routines	Students use pattern (mathematical procedure) to solve the problem
4	Narratives	Students make mathematical explanations to solve the problem

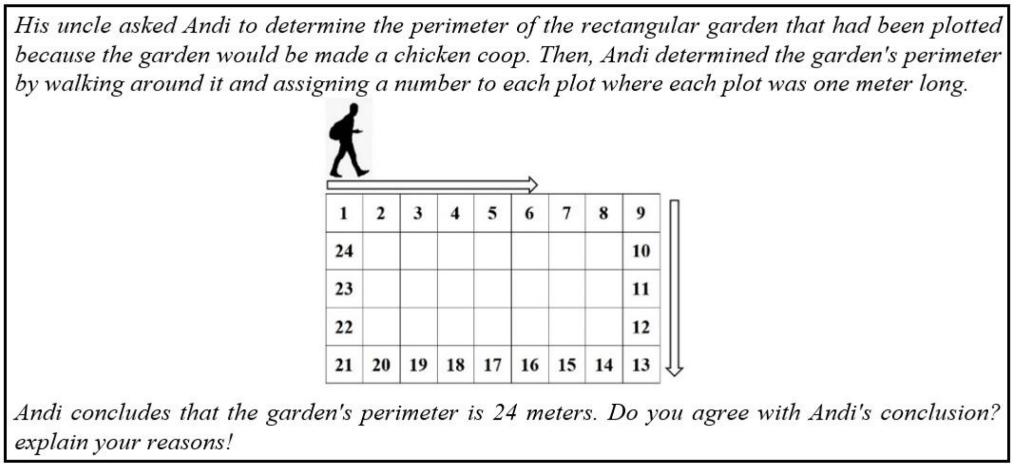


Figure 1. *Instrument*

This study took place at the junior high school in Madrasah Tsanawiyah (MTs) Surya Buana, located in Malang City, East Java Province, Indonesia. In the initial stage, the researcher collaborated with the mathematics teacher at the school to identify suitable participants. This collaboration aimed to select students who demonstrated strong critical thinking skills in mathematics and the ability to express their thinking clearly through oral communication. These abilities are needed to explore in depth students' cognitive processes. Furthermore, the mathematics teacher recommended students who were members of the school's mathematics olympiad team. According to the teacher, these students were considered to possess stronger critical thinking skills and oral communication abilities. The subjects of this study involved 17 students: three from Grade 7, five from Grade 8, and nine from Grade 9. Most of them were members of the mathematics olympiad group, indicating that they had above-average mathematical ability.

The data analysis in this study was conducted in several stages. In the first stage, initial coding was applied to the students' answer sheets. The coding component, including Interpretation (Int), Analysis (An), Inference (Inf), Word Uses (WU), Visual Mediators (VM), Narratives (N), and Routines (R). The coding was based on components of critical thinking skills and the commognitive indicators used by the students. This process resulted in two categories, namely Category A and Category B. Category A consists of students who met all components of critical thinking skills and commognitive indicators. Category B consists of students who met some components of critical thinking skills (interpretation) and some commognitive indicators (word use and visual mediators). One representative student from each category was selected for further analysis based on the completeness of the data. In the second stage, further coding was conducted on the think-aloud data and interview transcripts of the selected students. In the third stage, data consistency was examined across the answer sheets, think-aloud data, and interview transcripts. Based on data completeness and consistency, student 1 (S1) was selected to represent Category A and student 14 (S14) was selected to represent Category B. The research procedure presented in Figure 2.

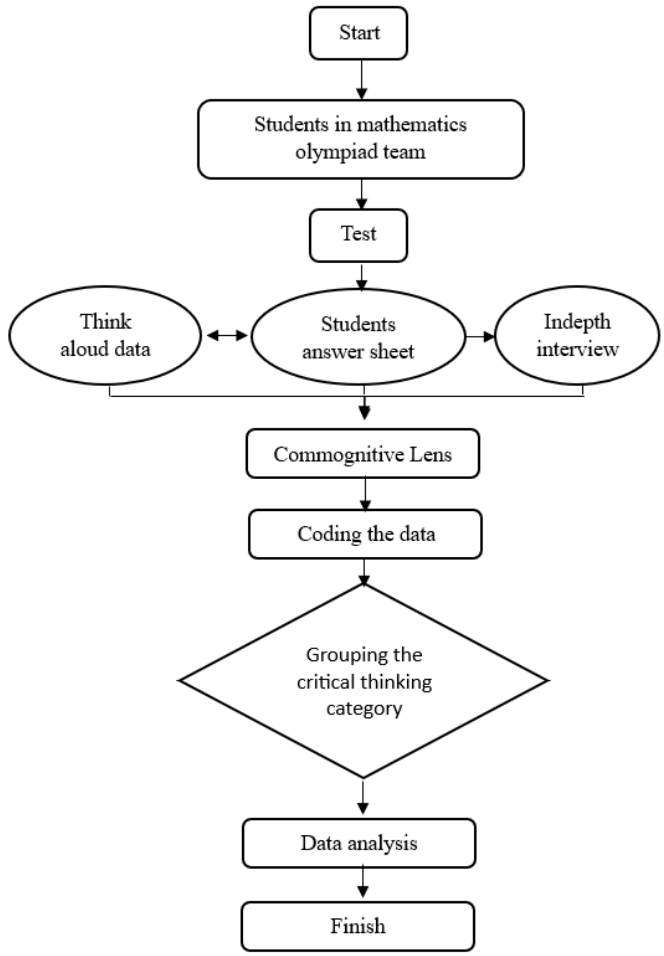


Figure 2. *The Research Procedure*

Results and Discussion

The results of this study describe students' thinking processes when solving cognitive conflict problems in geometry using commognitive framework. The term commognitive conflict is used to describe cognitive conflict situations within a commognitive framework. Based on the coding scheme outlined in the Methods section, the students were grouped into two categories (A and B) based on their critical thinking skills and commognitive indicators. One representative student from each category was selected for further analysis based on the completeness of the data. This section presents an in-depth description and analysis of two representative students from Categories A and B. Student 1 (S1) represents Category A, while Student 14 (S14) represents Category B.

1. Commognitive Conflict S1

S1 initially identified the perimeter of the garden through the following calculation: *"... 1, 2, 3, 4… , 8, 9 (counting while pointing at the plot), 9, 1, 2, 3, 4, 5 hmmmm….9..and 5 ...* ". Then, he formulates the perimeter of the rectangle, "… *(length + width) times two. ... 14 + 14 = 28…*". This excerpt was taken from S1's think-aloud protocol and indicates that the student employed conceptual understanding in his initial calculation.

However, S1 soon encountered a commognitive conflict when evaluating his solution in relation to the problem's context. He questioned himself: *"…is this going on inside or outside the plot, huh?"* This moment of hesitation prompted reflection, as he revisited the problem representation. After re-examining the visual, S1 exclaimed, *"Wow, this is going around the outside of the plot…"*, signaling a shift in his interpretation of the perimeter.

S1 then began to revise his approach by sequentially calculating the length and width based on the plot's boundaries rather than the number of individual squares. He clarified: *"…he should have reached the ninth square, so he should have started again from the ninth square to measure the width… so the perimeter is 28 meters, not 24 meters."* This adjustment reflects a refinement in S1's reasoning, moving from a misinterpretation to a more accurate conceptualization of perimeter within the given context.

The following excerpt from the interview with S1 provides insight into his cognitive processes.

R : *You seem to check the correctness of your calculations repeatedly. Could you explain that?.*

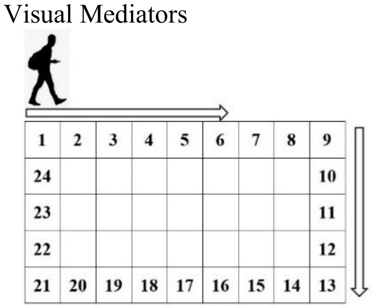
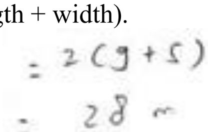
S1 : *I was trying to make sense of Andi's steps. I checked them again. Then I remembered how to calculate length and perimeter. So I figured the perimeter should be 28. I wondered why it didn't match. Eventually, I recalculated everything, trying to find where Andi made a mistake in tracing the garden's path. It turned out he had already passed the length of 9, and then continued from 10 when measuring the width. That's incorrect. The width should've started again from 9.*

This exchange illustrates how S1 actively engaged in evaluation and inference, two core components of critical thinking, by identifying inconsistencies, reassessing assumptions, and refining his understanding to align with correct mathematical reasoning. S1 proceeded to analyze the error in Andi's reasoning, noting that the length and width of the garden were not calculated correctly (indicating a missing segment). At this point, S1's reasoning reflects the analysis and evaluation stages of critical thinking, where he identified Andi's mistake as a miscalculation and articulated a conclusion based on his own cognitive process. During the explanation stage, S1 employed deductive reasoning, using the concept and general formula of a rectangle's perimeter to justify his answer. He meticulously re-checked both his calculations and the conditions provided in the problem to ensure accuracy and consistency.

An interesting aspect worth highlighting is that S1 did not initially realize that Andi was using the garden plot as a reference for calculating the perimeter. In a follow-up interview, it became evident that S1 focused primarily on the omission of four garden plots. He stated, "*…there are four plots that Andi missed…*". While this observation was mathematically valid, it lacked suitable in terms of the context. This indicates that S1's reasoning was partly constrained by his own interpretation framework, which led to a commognitive conflict—a tension between his existing discourse and the contextual demands of the problem. A detailed description of S1's commognitive conflict is presented in Table 2.

The case of S1 illustrates how commognitive conflict can act as a productive stimulus in advancing students' mathematical reasoning (Nachlieli & Heyd-Metzuyanim, 2022; Thoma & Nardi, 2017). According to Sfard (2007), commognitive conflict arises when the student encounters a mismatch between his prior discourse (the standard formula for perimeter) and the structure of the problem which used garden plots as referents. The student's initial interpretation (grounded in conventional mathematical procedures) failed to capture the contextual nuance embedded in the problem's representation. Kadarisma et al. (2020) stated that students' misinterpretations were influenced by their level of abstraction ability. This dissonance also triggered cognitive tension (Watson, 2007), prompting S1 to engage more deeply in critical thinking processes, particularly in the stages of analysis, evaluation, inference, and explanation.

Table 2. *S1 Commognitive Conflict in Solving Cognitive Conflict Problems in Mathematics*

Commognitive Conflict	Solution of Commognitive Conflict
S1 experienced a commognitive conflict while evaluating his solution within the context of the problem. The confusion arose from a discrepancy between his conceptual understanding and the procedural steps problem had applied.	S1 performs an in-depth and persistent problem analysis.
• Word Uses "*…is this going on inside or outside the plot, huh*?..."	• Word Uses *"... Wow, this is going in the outside plot…"* "*…there are four plots that Andi missed…*"
• Visual Mediators 	• Visual Mediators Picture of of cognitive conflict problems in geometry and his answer. Formula of rectangle circumference = 2 (length + width). 
• Routines S1 routinely performs addition operations from the concept of the perimeter of a rectangle.	• Routines S1 routinely checks the truth of the reasoning that is done.
• Narratives 2 (length + width) = length + width + length + width	• Narratives S1 recognizes Andi's error which results in a perimeter of 24 m. In calculating the circumference must fully determine the length and width.

The think-aloud data from S1 also demonstrates several elements of critical thinking. At the interpretation stage, S1 correctly applied the conceptual understanding of a rectangle. However, during the analysis stage, he encounters cognitive dissonance due to inconsistencies in his results, which initiated further reflection (Moustapha-Corrêa et al., 2021; Presmeg, 2016). According to Setiyani et al. (2025), reflection is carried out by carefully identifying key information in order to generate possible solutions. He critically evaluated his previous conclusion (that the perimeter is 24 meters) by reexamining the problem and verifying the accuracy of his reasoning.

Based on Sfard's commognitive framework, this moment represents a shift in discourse, wherein the student's mathematical thinking undergoes restructuring as a result of conceptual tension (Sfard, 2007). S1's use of deductive strategies, his efforts to resolve ambiguity in the problem, and his critical re-evaluation of the solution path indicate that his mathematical routines and reasoning narratives were undergoing refinement. Although S1 initially applied the correct perimeter formula, he was compelled to adjust his reasoning to accommodate the contextual representation of the garden (the layout of the plots). According to Thoma & Nardi (2017) this adaptation involved reinterpretation of visual mediators and adjustment of the terminology employed in the problem, reflecting the evolving nature of his mathematical discourse. Such instances illustrate the dynamic interaction between formal mathematical knowledge and contextual understanding (a core feature of commognitive conflict and its resolution).

2. Commognitive Conflict S14

S14 began her interpretation by applying the concept of perimeter that she had previously learned. Her initial strategy involved visually tracing the outline of the given object to estimate its perimeter. She expressed her reasoning by stating, "*... because it surrounds, so I answered agree because Andi surrounds his garden...*". This response indicates that S14 initially relied on an intuitive understanding of the concept of "surrounding" to justify her answer. According to Commognitive, this illustrates a breakdown between word use and visual mediators: while the word "length and width" triggered a procedural association with formulas, the visual mediator (a grid of garden plots) required her to interpret the physical layout rather than merely apply a rule. Eventually, she decided to follow Andi's method by tracing the plot's outline to find the perimeter.

However, as the task progressed, S14 encountered a commognitive conflict, a situation in which her existing discourse clashed with the demands of the problem. This was evident in her expression of confusion and hesitation: *"... I'm confused about it… Usually, if I calculate the perimeter, I can input length and width into the formula, but this is only given 1 meter for the length of 1 plot…*". Her confusion indicates that the familiar routines she had internalized (such as applying the standard formula for perimeter) were not immediately applicable in the new context, prompting her to reassess her approach. This confusion continued as she emphasized, "*Not only the perimeter is known… There should be known the length and width*.". This statement reflects her strong reliance on symbolic procedures rather than conceptual flexibility.

At the evaluation stage, the subject assesses the validity of her own reasoning based on prior understanding. These stages are illustrated in the following excerpt from the interview:

S14 : *I'm confused about the problem.*

R : *What are you confused about?*

S14 : *…. Usually, if I look for the perimeter of a rectangle, it's 2 (length + width), but that (while looking back at the problem) hmm, the length and width are actually known.*

This excerpt reveals a moment of internal contradiction. Earlier, the subject had stated that the problem did not provide the necessary dimensions (She said "*Not only the perimeter is known... There should be known the length and width*."). This inconsistency is indicative of a commognitive conflict, where her established discourse routines failed to immediately reconcile with the representations in the task. The conflict arose from her initial inability to translate the visual representation (the grid of plots) into numerical values that aligned with the formula she relied on.

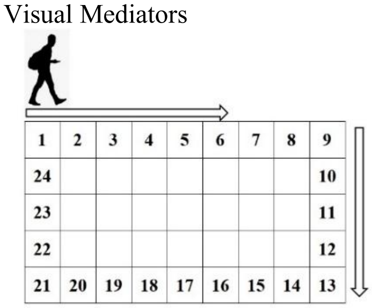
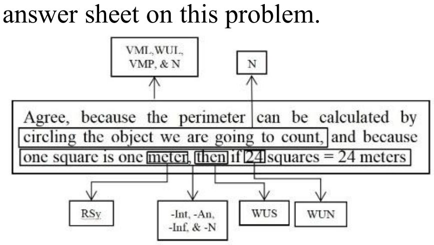
Prompted by the researcher to reconsider her approach, the subject paused to reflect, then revised her reasoning:

"*Oh yes, yes, yes... this side (long side) plus this side (short side)... Oh yes, the length is 9, and the width is 5. So, 2(9 + 5) = 28*".

This moment captures that commognitive conflict can act as a trigger for shifts in students' mathematical discourse. Gradually, S14 acknowledged the misalignment between her initial interpretation and the visual representation, leading her to restructure her discourse. The subject reflects her metacognitive awareness in monitoring and adjusting her reasoning process. Although her reasoning process was not flawless, it reflected progress toward deeper conceptual understanding through discourse transformation. Eventually, the subject successfully evaluated her earlier interpretation, identified the inconsistency, and inferred the correct dimensions based on the visual structure of the problem. The detailed manifestations of S14's commognitive conflict are presented in Table 3.

The case of Student 14 (S14) provides a compelling example of how commognitive conflict can foster shifts in mathematical discourse. Her reasoning process reveals the interaction among the four central components of commognition: word use, visual mediators, narratives, and routines. Initially, S14 relied heavily on procedural conventions, applying memorized formulas without adequately engaging with the visual context. She exhibited errors in interpreting and conceptualizing the perimeter of a rectangle. Umam et al. (2022) categorized critical thinking errors into four types: interpretive, conceptual, procedural, and technical. In S14's case, the difficulties were primarily interpretive and conceptual. This finding aligns with Pratiwi et al. (2022), who documented students' routine errors when engaging with cognitive conflict problems involving improper fractions.

***Table 3**. S14 Commognitive Conflict in Solving Cognitive Conflict Problems in Mathematics*

Commognitive Conflict	Solution of Commognitive Conflict
S14 experienced a commognitive conflict when the problem components could not be input into the formula for the rectangle perimeter. In this situation, S14 focuses on procedural understanding in finding the perimeter.	The researcher prompted S14 to reconsider possible alternative strategies, which led her to recognize the error in her initial reasoning.
• Word Uses "… *Usually, if I calculate the perimeter, I can input length and width into the formula*..." "*...There should be known the length and width*..." "*but this is only given 1 meter for the length of 1 plot*..."	• Word Uses "*...Oh yes, yes, is known the length and width ... So , 2 (9+5) = 28….*"
• Visual Mediators  The following is the coding of S14 answer sheet on this problem. 	• Visual Mediators Figure of cognitive conflict problems in geometry.
• Routines S14 relied on procedural routines for calculating the perimeter: 2(length + width).	• Routines S14 verifies her thought process.
• Narratives Must be known the length and width then input into the formula below. perimeter = 2 (length + width)	• Narratives S14 acknowledged the misalignment between her initial interpretation and the visual representation, leading her to restructure her discourse. Perimeter = 2 (length + width) = length + width + length + width

According to Sfard's theory (2007), this scenario highlights a breakdown between word use and visual mediators. While the phrase "length and width" activated procedural associations with formulas, the visual mediator (a grid of garden plots) required S14 to interpret spatial layout rather than apply a rule mechanically. This dissonance led to a commognitive conflict (Cooper & Lavie, 2021; Nachlieli & Heyd-Metzuyanim, 2022; Thoma & Nardi, 2017). Her engagement with visual elements (the garden layout), mathematical terminology, and the personal explanations she formulated collectively contributed to a gradual transformation in her problem-solving approach. Sfard (2007) asserted that the transformation from rigid procedural application to context-sensitive interpretation reflects a movement from ritualized to substantiated routines (signifying increased maturity in her mathematical discourse as articulated by Sfard).

Conclusion

The commognitive conflicts experienced by the subjects from each category highlight pivotal moments of cognitive engagement and discourse transformation. When confronted with cognitive conflict problems, the subjects initially relied on their prior knowledge and routine procedures. However, the emergence of conceptual discrepancies triggered confusion, prompting a shift from procedural reasoning to critical reflection and deeper analytical thinking. This shift activated various dimensions of critical thinking, including interpretation, analysis, evaluation, and inference. S1 and S14 do not represent all individual variations within each group. Instead, they serve as analytical representatives of Categories A and B based on clearly defined criteria. They were selected due to the completeness, consistency, and richness of their data across answer sheets, think-aloud protocols, and interview transcripts.

S1 represent Category A by demonstrating persistent re-evaluation, clear identification of the source of cognitive conflict, and efficient resolution through discursive validation, which reflects full alignment with critical thinking components and commognitive indicators. In contrast, S14 represent Category B by exhibiting interpretive and conceptual difficulties, particularly in reconciling formal procedures with visual mediators. Her gradual reconstruction of understanding during guided reflection illustrates partial fulfillment of critical thinking components and commognitive indicators. These findings illustrate how commognitive conflict can catalyze cognitive development by fostering shifts in mathematical discourse. The transformation from ritualized to substantiated routines (as described in Sfard's framework) was evident in both cases and underscores the value of designing learning experiences that challenge students' established patterns of thinking. Based on the result of this study, it is recommended that future research explore the development of mathematics instructional designs in cognitive conflict problems based on commognitive framework.

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