Volume 8, Nomor 1, Juli 2026, pp. 21-39

RANGE : Jurnal Pendidikan Matematika
e-ISSN :2685-2373

Unveiling Students' Cognitive Obstacles in Constructing Quadratic Function Representations: An APOS Theory-Based Analysis

Hesty Marwani Siregar [1], Nurjanah [2]*, Darhim [3], Turmudi [4]
[1,2,3,4] Department of Mathematics Education, Universitas Pendidikan Indonesia, West Java, Indonesia
*Correspondence: nurjanah@upi.edu

Received: September, 8th 2025. Accepted: May 26th 2026. Published: July, 23rd 2026

ABSTRACT

Quadratic functions are a fundamental topic in the high school curriculum, yet many students struggle to construct and coordinate symbolic, visual, and verbal representations. While previous studies have primarily focused on identifying procedural and conceptual errors, limited attention has been given to how these difficulties emerge from students' cognitive constructions. This study addresses this gap by employing APOS theory (Action, Process, Object, Schema) to analyse students' cognitive obstacles in constructing quadratic function representations. Using a descriptive qualitative approach supported by descriptive quantitative analysis, this study collected data from 35 tenth-grade students in Pekanbaru through a mathematical representation test and semi-structured interviews. The findings reveal a dominant cognitive pattern in which most students remained at the Action and Process stages, with only a few reaching the Object stage and none achieving Schema. This indicates that students' reasoning tends to break down during the transition from procedural execution to conceptual understanding, resulting in fragmented representations that are not yet meaningfully integrated. This study contributes theoretically by demonstrating how cognitive obstacles are not merely errors, but indicators of incomplete mental construction across APOS stages. It offers a more explanatory perspective on students' difficulties than studies that focus mainly on observable errors. Practically, the findings highlight the need for more strategic instructional support, particularly through transition-based approaches that help students move across APOS stages and integrate multiple representations more coherently. These insights offer a conceptual basis for improving the teaching and learning of quadratic functions.

Keywords: APOS, cognitive obstacle, mathematical representation, quadratic function.

How to Cite: Siregar, H. M., Nurjanah, Darhim, & Turmudi. (2026). Exploration of students' cognitive obstacles in constructing mathematical representations based on APOS theory as the foundation for quadratic function learning strategies. *Range: Jurnal Pendidikan Matematika*, 8(1), 21-39.

Introduction

Quadratic functions are essential topics in the high school mathematics curriculum that play a strategic role, both theoretically and practically. In line with this, several studies emphasise that quadratic functions are a fundamental concept for mastering key algebraic ideas necessary for the study of calculus and modelling in both pure and applied contexts, and they constitute an essential part of algebra or introductory mathematics courses at the university level (Saputra et al., 2018; Wilkie, 2024). However, despite their importance, students often struggle to connect algebraic and graphical representations and experience difficulties in interpreting function concepts, while also making errors across the processes of formulating, employing, and interpreting mathematical solutions (Hidayanti & Hidayati, 2025; Ly &

Baba, 2023; Ruli, Priatna, Prabawanto, & Mulyana, 2018). These difficulties indicate that students' challenges are not only procedural but are also related to deeper cognitive obstacles that hinder their ability to develop coherent mathematical understanding.

One important source of these difficulties lies in students' limited ability to work with multiple mathematical representations. A comprehensive understanding of quadratic functions requires proficiency in symbolic, visual, and verbal representations, as well as the ability to shift between these modes to foster conceptual insight and adaptive problem-solving. The use and interconnection of different representations, such as symbolic, graphical/visual, and verbal, and the ability to transition between them, are identified as key activities in constructing conceptual understanding of functional relationships and in acquiring more flexible problem-solving strategies (Testa & Catena, 2022; Zentgraf & Prediger, 2024). However, empirical studies consistently show that students experience significant difficulties when transforming quadratic functions across representations, such as from symbolic to graphical or from symbolic to verbal forms. These difficulties are evident in students' struggles to translate between representations and to express functional relationships accurately (Daher & Anabousy, 2015; Rahmawati, Sa'dijah, & Sudirman, 2025; Ramírez, Cañadas, & Damián, 2022; Wibawa, 2019). These difficulties indicate that the issue is not merely a lack of procedural knowledge, but reflects deeper cognitive barriers in coordinating and interpreting representations.

Challenges in learning quadratic functions, such as difficulties in graph representation, determining maximum/minimum values, and understanding equations, are consistent with findings that the most frequent errors involve process, comprehension, carelessness, conceptual understanding, principles, and calculations (Hoon et al., 2018; Azizah et al., 2021; Rahman & Foad, 2021). These difficulties are not isolated procedural issues but can be interpreted as indications of underlying cognitive barriers, particularly students' failure to coordinate and transition between different representations of quadratic functions. Such patterns suggest that students experience obstacles in organizing mathematical processes and meanings, thereby highlighting the need for a theoretical framework that can systematically explain the nature and progression of these difficulties.

These findings indicate the presence of cognitive obstacles. Originating from Gaston Bachelard's ideas, the concept has since been adopted by scholars across various disciplines. A cognitive obstacle refers to a barrier in reasoning that emerges when strategies or knowledge that typically enable students to solve problems prove insufficient in different contexts. According to Kartinah et al. (2020), cognitive obstacles are forms of knowledge that work effectively in specific problems but fail when students face different problem situations. Antonijević (2016) emphasised that when students overcome a cognitive obstacle in mathematics learning, the process becomes a bridge that leads them to new knowledge that was previously unfamiliar or not understood. Chew & Cerbin (2020) emphasised that a lack of effort does not always cause learning failure; even capable students can experience it. Therefore, identifying cognitive obstacles not only serves to map learning difficulties but also provides a basis for teachers to design more targeted instructional interventions to develop various mathematical abilities, particularly mathematical representation skills comprehensively.

Cognitive obstacles are closely linked to students' mental processes and can be examined using theories that account for the mechanisms of mental construction in learning. One relevant theory is the APOS theory (Action, Process, Object, Schema) developed by Dubinsky. This theory views mathematics learning as a process of mental construction that progresses through four main stages. Students perform explicit mathematical manipulations based on specific instructions or algorithms at the Action stage. This stage then develops into a Process stage, when students perform such manipulations internally without always relying on explicit procedures. Next, in the Object stage, students can treat a process as a complete and flexible entity. Finally, the entire structure of understanding culminates in a Schema, a coherent network of knowledge that supports the understanding and solution of various mathematical situations. However, research shows that transitions between these stages do not always occur smoothly; for instance, students may perform actions well but fail to internalise them into processes, creating new cognitive obstacles (Nisa, Waluya, Kartono, & Mariani, 2019).

Within quadratic functions, APOS theory provides a comprehensive framework for pinpointing where cognitive obstacles emerge and informing strategies supporting transitions between stages. This theory emphasises knowledge construction and offers guidance for designing instructional interventions that promote students' mental development (Tatira, 2022). As highlighted by Dubinsky and McDonald, effective instruction depends on facilitating the construction of appropriate mental structures and guiding students in processing them to achieve meaningful conceptual understanding (Syamsuri & Marethi, 2018). Recent developments in APOS research further highlight that students' difficulties are not merely procedural, but are often rooted in challenges in transitioning between cognitive stages, particularly from action to process and from process to object (Oktaç, 2022). Empirical studies consistently show that students frequently fail to construct coherent mental structures, resulting in fragmented understanding of mathematical concepts (Adamura, Juniati, & Khabibah, 2026; Koyunkaya & Boz-Yaman, 2023; Osorio, Bermúdez, & Jiménez, 2026). In the context of quadratic functions, these difficulties can be interpreted as manifestations of incomplete cognitive development, where students are unable to coordinate and connect multiple representations effectively (Yerizon, Sukestiyarno, Arnellis, Suherman, & Hevardani, 2024).

Studies on students' difficulties with quadratic functions have mostly focused on procedural and conceptual errors rather than cognitive obstacles based on APOS stages and their relation to mathematical representations. Previous research has emphasized error analysis using other frameworks. For example, Islamiyah et al. (2023) identified five types of errors in solving word problems based on Nolting's theory. Akhsani & Nurhayati (2020) found that students' graphing errors were dominated by procedural mistakes reducible through PBL with GeoGebra. Safitri et al. (2025) reported that students across achievement levels struggled to translate symbolic forms into graphs, with interpretation, implementation, and preservation errors. While previous studies have identified various cognitive obstacles and errors in quadratic functions, such as procedural mistakes or difficulties in connecting representations, they have primarily described the types of obstacles without investigating the underlying mental processes that lead to their emergence. In other words, the question of how students' thinking becomes stalled and why their knowledge fails to transfer across contexts has not been systematically addressed. A systematic search in Scopus and Google Scholar (up to September 2025) using the combined keywords "APOS" AND "quadratic function" AND "cognitive obstacle" AND "mathematical representation" found no articles that specifically applied the APOS framework to investigate students' cognitive obstacles in constructing mathematical representations of quadratic functions. Although cognitive obstacles have been analysed in other mathematical domains, such as definite integrals (Kartinah, Nusantara, Sudirman, & Daniel, 2021), the absence of studies applying APOS to quadratic functions indicates a clear gap. Therefore, this study positions itself to fill that gap by examining not only what obstacles occur, but also how students' mental constructions—mapped through APOS stages—give rise to those obstacles.

Therefore, the novelty of this study lies in integrating APOS theory to identify students' cognitive obstacles in constructing mathematical representations of quadratic functions. Unlike previous studies that primarily describe observable errors, this study focuses on the underlying mental construction processes that lead to those errors, particularly across the Action, Process, Object, and Schema stages. By mapping students' difficulties within this cognitive framework, the study not only extends the application of APOS theory in mathematics education but also provides a more explanatory perspective on how representational failures emerge. In addition, this approach offers a pedagogical foundation for designing more adaptive instructional strategies that explicitly support students' transitions between cognitive stages.

Building on this background, this study aims to describe students' cognitive obstacles in mathematical representation skills related to quadratic functions, framed by the APOS stages. Theoretically, this study is expected to enrich the application of APOS theory in analysing learning difficulties in mathematics, focusing on cognitive obstacles. Practically, we expect the findings to provide recommendations for instructional strategies that help students overcome cognitive obstacles and integrate various forms of representation coherently.

Methods

This study employed a descriptive qualitative approach. This approach is relevant when researchers aim for a comprehensive description of an event, thereby presenting research realities closely aligned with participants' experiences (Villamin, Lopez, Thapa, & Cleary, 2024). In this context, the research explores students' cognitive obstacles in constructing mathematical representations of quadratic functions. The descriptive qualitative approach is relevant as it enables researchers to trace students' thought processes more comprehensively, including identifying how students interpret problems, their strategies, and why difficulties occur at certain stages. This study adopts a qualitative research paradigm with descriptive quantitative support. While the primary focus is on exploring students' cognitive obstacles through in-depth qualitative analysis, descriptive statistics are used to provide an overview of students' performance across different forms of representation. This integration does not aim to generalize findings but to strengthen the interpretation of qualitative data.

The research subjects were 35 tenth-grade students at a public senior high school in Pekanbaru. All students in the class were included as research subjects because they had studied quadratic functions according to the curriculum, thus having the academic foundation to complete the test designed by the researchers. The lesson on quadratic functions began with conceptual explanations and was followed by group problem-solving activities, supporting both individual and collaborative learning. All students completed the mathematical representation test created by the researchers. From the 35 students, 6 students were purposively selected as interview participants. They were chosen to represent three achievement groups (high, medium, and low), with two students from each group, in order to capture diverse answer patterns and cognitive obstacles. The selection of 35 students provides sufficient variation in responses to capture diverse cognitive patterns, while the in-depth analysis of six interview participants allows for a detailed exploration of students' mental constructions across different ability levels.

The research instruments included a mathematical representation test and a semi-structured interview guide. The mathematical representation test consisted of three essay questions, each representing different forms of representation: visual, symbolic, and verbal. Specifically, the first question (visual representation) asked students to draw the trajectory of a skateboard shaped like a parabola, based on a quadratic function. The second question (symbolic representation) required students to model the trajectory of a ball kicked toward a goal into a quadratic function and determine whether the ball would enter. The third question (verbal representation) asked students to interpret the meaning of the zeros of a quadratic function in a business profit context. The test instrument underwent content validation by five experts: mathematics education lecturers, language lecturers, and experienced mathematics teachers. The validation results indicated that the test was feasible to use with minor revisions, particularly regarding clarity of wording. In addition to the test, another instrument used was the semi-structured interview guide. The researchers explored the reasons behind students' answers and obtained a deeper insight into their thought processes through interviews.

The researchers organised data collection in two main stages during the second semester of the 2024/2025 academic year. The first stage involved administering the mathematical representation test to all students in the research class. Students completed the test within two class hours, after which the researchers collected and analysed their responses. Following the completion of grading, the researchers initiated the second stage one week later, involving semi-structured interviews with students selected based on the diversity of their test responses. The interviews were conducted face-to-face in one day, recorded with the students' consent, and then transcribed for further analysis.

Data analysis followed Miles and Huberman's interactive model, which involves data reduction, data display, and conclusion drawing (Asipi, Rosalina, & Nopiyadi, 2022). This model was selected because the study aims to give a detailed, descriptive account of students' cognitive obstacles at each APOS stage. The interactive cycle of reducing, organizing, and visually displaying data provides a systematic way to capture the complexity of students' written responses and interview transcripts and to trace how their mathematical representations reveal cognitive obstacles. Compared with other qualitative analysis frameworks such as thematic analysis or grounded theory, which focus on generating new theory, the Miles and Huberman model is more appropriate for a descriptive qualitative design that seeks to map and interpret students' thought processes in detail rather than to build new theoretical constructs

Guided by this framework, data analysis focused on mapping students' responses into APOS stages to identify corresponding cognitive obstacles. Students' written answers were first examined to determine their level of representation ability (visual, symbolic, and verbal), supported by descriptive statistics to provide an overview of performance. Subsequently, each response was analysed using an APOS-based coding scheme to identify whether students operated at the Action, Process, Object, or Schema stage. The identification of cognitive obstacles was then conducted by examining inconsistencies, incomplete reasoning, and failures in transitioning between stages. Interview data were used to validate and deepen the interpretation of students' cognitive processes, ensuring that identified obstacles reflected actual thinking patterns rather than surface-level errors.

To map student responses into APOS stages, a coding scheme was developed based on Dubinsky's framework and adapted to the context of quadratic functions. Responses were classified as No Answer when students left the task blank or failed to activate prior knowledge. The Action stage was assigned when students applied direct procedures without deeper conceptualization. The Process stage was identified when procedures were internalized but still accompanied by misconceptions or incomplete understanding. The Object stage was assigned when students treated a process as a complete entity, such as recognizing a quadratic function as a whole or attributing meaning to its elements. Finally, the Schema stage referred to integrated reasoning across different processes and representations. After identifying the APOS stage, the researchers determined the cognitive obstacles associated with each stage. To reinforce the identification of these obstacles, the researchers used interview data in which students explained their thought processes. To ensure the reliability of the APOS coding, the researchers worked collaboratively in reviewing and discussing how students' responses were classified into each APOS stage. The coding process was carried out iteratively, with initial interpretations being compared and refined through ongoing discussions until a shared understanding was reached. This collaborative approach helped maintain consistency in interpreting students' cognitive processes and strengthened the dependability of the findings.

Following Lincoln and Guba's framework, this study addressed data trustworthiness through the four criteria of credibility, transferability, dependability, and confirmability (Stahl & King, 2020). The researchers validated the test instrument with five experts and applied methodological triangulation by comparing data from written tests and interviews to maintain credibility. For transferability, the researchers provided a comprehensive description of the study, encompassing the school context, number of students, class characteristics, and research procedures. The researchers maintained consistency in the research process to ensure dependability. The researchers kept all raw data from students' answer sheets and interview transcripts to retrace the process and evaluate stability. Furthermore, the data analysis procedures were systematically explained and discussed with the academic supervisor to ensure consistency of interpretation. Confirmability was maintained by presenting empirical evidence supporting each interpretation, in the form of excerpts from students' answers and interview quotations, and by engaging in discussions with the academic supervisor and mathematics teacher to minimise researcher bias. The overall procedure is summarised in Figure 1 as a flow chart to illustrate the sequence from instrument development to data analysis.

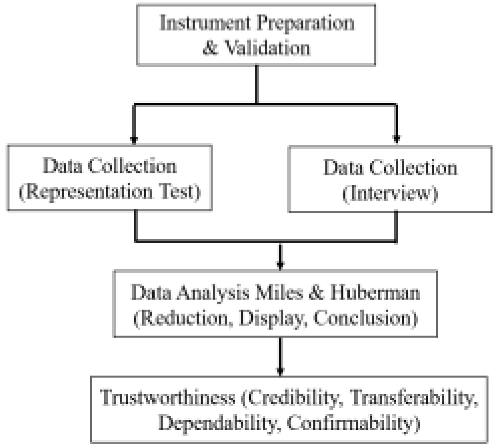


Figure 1. *Research Procedure Flow Chart*

Results and Discussion

The presentation of research findings begins with analysing students' achievement on each indicator of mathematical representation ability, including visual, symbolic, and verbal representation. Among the three indicators, visual representation showed relatively better achievement than symbolic and verbal, with 25%, although still not optimal. Conversely, the lowest achievement was found in verbal representation at 9%, with achievement in symbolic representation also relatively close at 10%. These findings are consistent with Shinariko et al. (2021), who reported that students' mathematical representation ability in quadratic functions tends to be low, particularly in symbolic and verbal representations, while visual representation, although relatively better, still falls into the intermediate category. The low achievement in all three indicators indicates that students have not fully mastered the mathematical representation of quadratic functions, thus requiring further analysis through the APOS framework to identify at which stage their thought processes encountered obstacles. Table 1 presents the distribution of APOS stages and the cognitive obstacles experienced by students. Students' answers were categorised into APOS stages based on predetermined criteria. These criteria, along with examples of typical student responses in the context of quadratic functions, are presented in Table 1.

Table 1. *Criteria for Categorizing Students' Responses Based on APOS*

APOS Stage	Response Criteria	Example Indications in the Context of Quadratic Functions
No Answer	Blank answer or simply copying the problem statement	The answer sheet is left blank; rewriting the problem statement without any attempt to solve it
Action	Solving the problem mechanically/partially without conceptual understanding	Drawing a parabola in a "U" shape without considering key points of the graph; writing down a formula but not continuing the calculation and not knowing why the formula was chosen
Process	Beginning to give meaning to procedures, but still showing misconceptions or incomplete reasoning	Drawing the graph by determining some elements (vertex, intercepts) but failing to understand their

		relationships, leading to misconceptions or unfinished solutions
Object	Treating the process as a whole entity and showing conceptual understanding of quadratic function elements	Correctly selecting the formula to be used, but struggling to interpret the obtained result within the problem context
Schema	Integrating various processes and representations (visual, symbolic, verbal) consistently to solve the problem	Connecting algebraic form, parabola graph, and verbal interpretation in real-life contexts coherently

Based on these criteria, the distribution of students' responses for each representation indicator was obtained. The results of this categorisation are presented in Table 1.

Table 2. *Distribution of Students' Responses Based on APOS Stages*

Indicators of Mathematical Representation Ability	Number and Percentage of Students					Cognitive Obstacles
	No Answer	Action	Process	Object	Schema	
Visual	8 (23%)	11 (31%)	16 (46%)	0	0	1. Students could not transfer their knowledge of quadratic functions to the problem context (No Answer). 2. Drawing a parabola without calculation, but the points plotted were incorrect (Action). 3. Determining one or two elements (vertex, x-intercepts, y-intercept) without understanding their use in graph construction (Action). 4. Misconception of algebraic operations involving radicals (Process). 5. Misconception of factorisation (Process). 6. Limited understanding of the real roots of quadratic equations (Process). 7. Limited understanding of quadratic function graphs (Process).
Simbolic	19 (54%)	8 (23%)	5 (14%)	3 (9%)	0	1. Inability to relate problem information with prior knowledge (No Answer). 2. Using procedures without considering the suitability of the problem data (Action). 3. Limited understanding of the quadratic function formula when the vertex is given (Process). 4. Limited understanding of the meaning of points lying on the graph of a quadratic function (Object). 5. Limited ability to interpret real-life contexts into mathematical models (Action, Process, Object).
Verbal	29 (82%)	2 (6%)	2 (6%)	2 (6%)	0	1. Students could not transfer their knowledge of quadratic functions to the problem context (No Answer). 2. The procedure is executed to determine the roots of the function, but without

		assigning meaning to the result (Action). 3. Misconception of the meaning of roots, namely as the boundary before the function becomes zero (Process). 4. Limited ability to interpret the roots in real-life contexts (Object).

The analysis results in Table 2 show that students' cognitive obstacles in constructing mathematical representations of quadratic functions appear in every indicator, with a tendency for students to remain at the initial stage of APOS. This finding is consistent with Bete (2019), who reported that students across ability levels continued to make conceptual, principled, and careless errors when solving algebra problems. A substantial proportion of students provided no answer, indicating a pre-Action cognitive obstacle, where students were unable to activate prior knowledge or initiate the problem-solving process. However, to analyse cognitive development within the APOS framework, the determination of dominant stages focuses on students who attempted the tasks. Based on this consideration, Table 3 summarises the dominant APOS stages and key cognitive obstacles across representations.

Table 3. *Summary of Cognitive Obstacles Across Representations*

Representation	APOS Stage	Key Cognitive Obstacles
Visual	Action-Process	Intuitive graphing, misconceptions of roots and discriminant.
Symbolic	Action-Object	Failure to connect context to formula, incomplete parameter understanding.
Verbal	Action-Object	Inability to interpret the meaning of roots in context

The visual representation problem asked students to draw the trajectory of a skateboard shaped like a parabola. The most prominent obstacle appeared at the initial stage, where many students left the answer blank, indicating difficulty in activating prior knowledge and connecting contextual information with the quadratic function graph. Among those who attempted the problem, most students remained at the Action and Process stages. At the Action stage, students tended to draw parabolas intuitively or based on limited elements such as the vertex or intercepts, reflecting a superficial understanding of the graph as merely a "U-shape." As their reasoning developed into the Process stage, more conceptual difficulties became evident, including errors in factorisation, misconceptions about roots, and a lack of awareness that some quadratic equations do not have real roots. At this point, no students demonstrated Object-level understanding, as they were unable to integrate algebraic and graphical elements into a coherent representation. These findings align with prior research showing that students struggle to convert between representations, often perceive parabolas merely as U-shaped figures rather than functional relationships, and face persistent difficulties in algebraic manipulation of quadratic equations (Gagatsis et al., 2016; Memnun et al., 2015; Tendere & Mutambara, 2020). An example of a student's answer, containing errors in factorisation and graph elements, is presented in Figure 2.

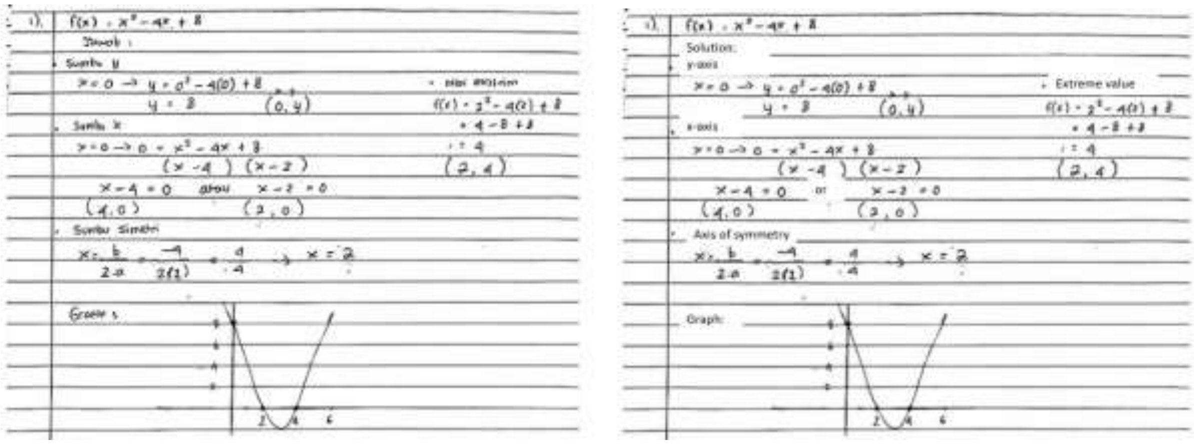


Figure 2. *Example of Student Answer to Visual Task*

Figure 2 shows that the student attempted to factorise $x^2 - 4x + 8$ into $(x - 4)(x - 2)$, then used the result to determine the X-intercepts. This mistake affected the graph drawn, as the parabola did not match its algebraic properties. In the interview, the student stated, “When looking for the X-intercept, I was confused about finding two numbers that multiply to 8 and add to –4, but I couldn’t find them. Finally, I just chose 2 and 4. But when I drew it, the graph looked square, not like a parabola. Connecting it to the vertex (2,4) resulted in a triangle. So I just made a curve, because I remembered that the graph of a quadratic function is a parabola.” This statement indicates that the student had internalised the basic concept of a parabola but had not yet conceptualised the discriminant as an object explaining the graph’s properties. Furthermore, prior knowledge did not help the student understand that some quadratic equations have imaginary roots. Instead of realising that a discriminant < 0 means no X-intercepts, the student still attempted to use factorisation and drew a parabola based on the remembered general shape. This finding is consistent with Kabar (2018), who reported that students tend to associate their mental representations of quadratic equations primarily with factorisation.

Taken together, students’ visual representations show that their understanding is still mostly procedural and not well connected. Although they can recognise the general shape of a parabola, they struggle to link algebraic concepts with the graph in a meaningful way, which prevents them from moving from the Process stage to the Object stage. As a result, their graphs tend to be based on guesswork or partial knowledge rather than a clear understanding of how algebra and graphs are related.

The symbolic representation task asked students to analyse a story about the trajectory of a ball kicked toward a goal. Within symbolic representation, cognitive obstacles occurred throughout nearly all APOS stages, primarily at the initial stage. Most students did not answer, indicating an inability to link contextual information with the quadratic function they had already learned. At the Action stage, obstacles arose when students failed to select formulas appropriate to the problem’s data. For illustration, one student wrote an initial interpretation of the problem, but it was incorrect and stopped at an early stage. Figure 3 shows an example of the student's answer.

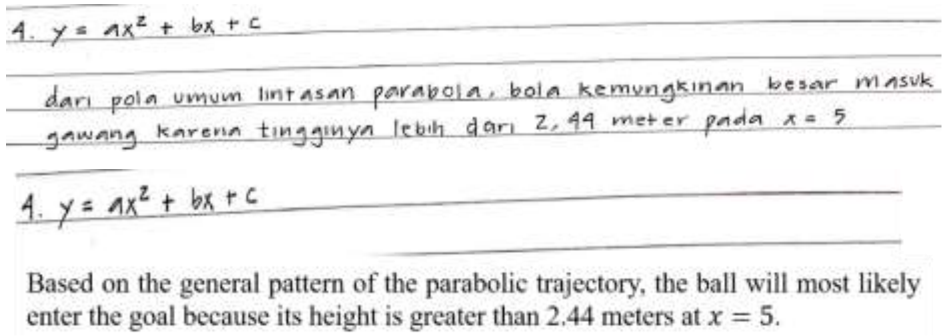


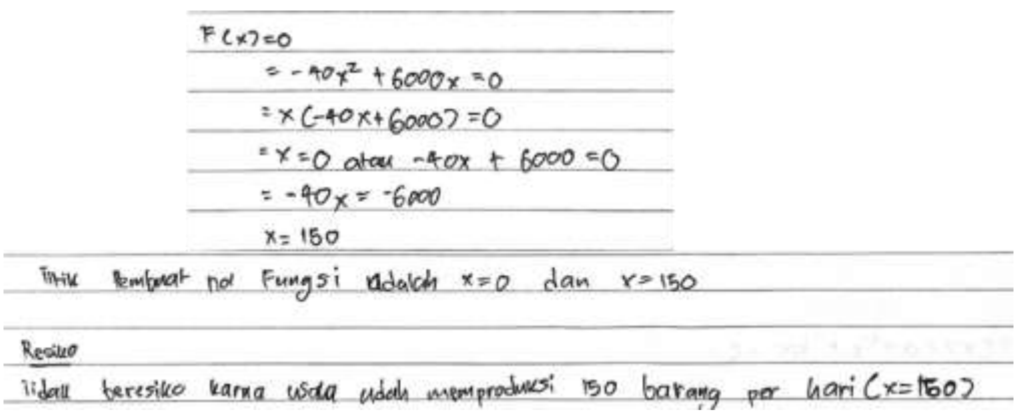
Figure 3. *Example of Student Answer to Symbolic Task*

When confirming the answer during the interview, some students were confused about interpreting the meaning of the ball's height relative to the goal; "…The ball should be higher ma'am, uh, lower… This problem asks whether the player can score a goal. So, the formula to find out whether someone can score, I don't know." These statements indicate that, despite having some understanding of the context, students were unable to link it consistently to the appropriate mathematical model. This obstacle demonstrates that cognitive obstacles are not only about the inability to compose procedures, but also about limitations in attributing meaning to symbolic results in real-life contexts. This type of obstacle aligns with findings that students often fail to transfer prior knowledge into word problems, thereby causing errors in both problem comprehension and solution selection (Bauk, Mamoh, & Simarmata, 2022).

At the Process stage, some students managed to write the quadratic function using the vertex. However, their work halted when they became confused about determining the value of a, which prevented them from completing the function. At the Object stage, some students did not understand the meaning of the points through which the quadratic function graph passes. Other students realised that they should substitute the goal distance into the function, but they failed to interpret the result correctly. For example, some correctly obtained the ball's height but incorrectly concluded that the ball had to be lower than the goal to enter. This obstacle highlights students' limitations in attributing symbolic results to real-life contexts. This finding is consistent with Díaz et al. (2020) and Ramírez et al. (2022), who reported that many students encounter deadlock at the outset of contextual problems due to forgetfulness or emotional barriers and commit procedural and conceptual errors stemming from insufficient conceptual understanding. In other words, although students know some algebraic procedures for quadratic functions, they cannot apply them appropriately in different contexts, since various mathematical expressions are not conceptually understood.

Overall, symbolic representation shows a clear gap between carrying out procedures and truly understanding the concepts behind them. Although some students reach the Object stage, their reasoning is still limited because they do not fully understand the meaning of the symbols they use. In many cases, students can apply formulas correctly, but they struggle to explain or interpret the results in a meaningful way, indicating that their understanding is still not fully developed.

The next problem concerned verbal representation. In the problem, students described the meaning of the quadratic function's zeros in the given context. The majority of students failed to provide an answer, as they were unable to link the quadratic equation to the given context. This obstacle indicates a limitation in transferring symbolic knowledge into meaningful verbal representation. In line with Nurrahmawati et al. (2021), many students struggle to maintain consistent meaning across symbolic and verbal representations. At the Action stage, some students could determine the function's zeros, but stopped at numbers without meaning, reflecting procedural and conceptual obstacles. At the Process stage, conceptual obstacles were evident when students realised that zeros mean zero revenue, but misinterpreted it as "a boundary before zero." Furthermore, at the Object stage, obstacles appeared in interpretative limitations: students knew that zeros meant the function equals zero, but thought the condition posed no risk, whereas it is actually a critical point. Figure 4 shows an example of the student's answer.


F(x)=0
$= -40x^2 + 6000x = 0$
$= x(-40x+6000) = 0$
$= x = 0$ atau $-40x + 6000 = 0$
$= -40x = -6000$
$x = 150$
Titik pembuat nol Fungsi adalah x=0 dan x=150
Resiko
Tidak beresiko karna usda udah memproduksi 150 barang per hari (x=150)

The zeros of the function are $x = 0$ and $x = 150$.

Risk
There is no risk because the business has already produced 150 items per day (x = 150).

Figure 4. *Example of Student Answer to Verbal Task*

Interviews revealed similar obstacles among students who reported a lack of understanding of the meaning of zeros. The findings reveal a disconnection between symbolic representation and its contextual meaning. These findings align with studies indicating that students often struggle to accurately interpret algebraic expressions (Erdem, Zengin, & Erdem, 2022). Thus, no student reached the schema stage, as complete integration between algebraic calculation, the meaning of zeros, the range of safe production, and risk of loss did not occur.

These findings indicate that verbal representation poses the greatest cognitive challenge for students. This is because it requires not only procedural and conceptual understanding, but also the ability to interpret mathematical meaning within a real-life context. Since these aspects are not yet well integrated, many students struggle to explain their results meaningfully, which ultimately prevents them from reaching Schema-level understanding.

Across the three representation types, students' cognitive obstacles appeared at nearly all APOS stages and prevented them from reaching Schema. Although visual representation was relatively better than symbolic and verbal representation, students' performance remained largely procedural, with only a few reaching the Process and Object stages. This overall pattern is consistent with Sholahudin et al. (2025), who found that students with stronger ability tend to shift more flexibly across representations, whereas those with lower ability experience greater difficulty. In this study, such difficulty was reinforced by students' limited awareness of their own misconceptions, leading them to repeatedly apply procedures that were inconsistent with algebraic properties or problem contexts.

These findings do more than confirm the persistence of cognitive obstacles in quadratic functions; they also explain where and why students' reasoning breaks down across representation types. Previous studies on quadratic functions have mainly described students' procedural, conceptual, or representational errors in problem solving, but they have not sufficiently explained the mental processes underlying those errors (Díaz et al., 2020; Islamiyah et al., 2023; Safitri et al., 2025). By applying APOS theory, this study addresses that gap by showing how students' reasoning stalled at different stages of mental construction across visual, symbolic, and verbal representations. Most students remained at the Action and Process stages, only a few reached the Object stage, and none achieved Schema. Thus, the novelty of this study lies in offering a stage-based cognitive explanation of representational difficulty, rather than merely identifying observable errors.

Theoretically, this study contributes to the literature by showing that students' representational difficulties in quadratic functions can be understood as stage-specific cognitive obstacles within the APOS framework. Rather than appearing as isolated procedural mistakes, these difficulties reflect incomplete transitions from Action to Process, from Process to Object, and ultimately the absence of Schema-level integration across visual, symbolic, and verbal representations. This perspective helps explain why students may be able to carry out a procedure in one form but still fail to interpret or connect it meaningfully to other representations.

Practically, the findings imply that instruction on quadratic functions should be designed as a transition-based APOS learning model. Since most students remained at the Action and Process stages, teaching should not stop at procedural practice, but should deliberately support students in progressing from carrying out procedures, to reflecting on their meaning, and finally to coordinating visual, symbolic, and verbal representations as integrated objects of thought. This can be facilitated through exploratory tasks, guided reflection, and integrative activities that require students to justify procedures, interpret results in context, and connect multiple representations. Such an approach is in line with Bicer (2021), who emphasised that students' mathematical understanding develops more strongly when they are encouraged to create, interpret, and connect different forms of representation. In this way, instruction can more systematically address students' cognitive obstacles and support the development of Schema-level understanding.

Conclusion

This study found that students' difficulties in solving quadratic function representation tasks did not appear as separate mistakes, but as a consistent pattern of cognitive barriers across APOS stages. Before reaching the APOS construction stages, some students were unable to activate prior knowledge and connect contextual information with relevant quadratic function concepts, which led to no response at all. At the Action stage, students were generally able to apply procedures, but their work tended to be mechanical, such as plotting graphs without valid calculations, identifying elements of a quadratic function without understanding their role, or using procedures without considering whether they matched the problem data. At the Process stage, students encountered more explicit conceptual obstacles, including misconceptions in algebraic operations, factorisation, the meaning of real roots, and the interpretation of quadratic graphs. At the Object stage, only a few students showed a more advanced understanding, yet they still experienced difficulty in assigning coherent meaning to mathematical results, especially when interpreting points, roots, and quadratic models in real-life contexts. This shows that representational failure in quadratic functions is closely related to students' incomplete progression from Action to Process, from Process to Object, and ultimately toward Schema.

The findings also highlight the conceptual value of APOS Theory in explaining students' representational difficulties. Rather than viewing students' errors only as incorrect answers or procedural weaknesses, this study shows that such difficulties can be understood as signs of incomplete mental construction. In this sense, the study contributes not only by identifying where students experience difficulty, but also by explaining why these difficulties persist across different forms of representation.

Practically, these results suggest that support for students should be designed more strategically around transitions between APOS stages. Learning activities should not only train students to apply formulas, but also help them activate relevant prior knowledge, use procedures meaningfully, interpret mathematical results conceptually, and relate one representation to another in order to gradually build more complete conceptual understanding. Therefore, future research may focus on developing and testing an APOS transition-based intervention model as a more explicit instructional framework to help students strengthen their representational understanding of quadratic functions.

Acknowledgement

This research was financially supported by *Lembaga Pengelola Dana Pendidikan Indonesia* (LPDP) – Indonesia Endowment Fund for Education Agency. I sincerely appreciate its contribution to the completion of this study.

References

Adamura, F., Juniati, D., & Khabibah, S. (2026). Profile of Students' Ability to Solve Group Material Problems Based on APOS Theory Reviewed from Differences in Mathematical Ability. *Journal of Science and Education (JSE)*, *6*(2), 1263–1275. https://doi.org/10.58905/jse.v6i2.737

Akhsani, L., & Nurhayati, E. (2020). The Error In Drawing Graphic Of Quadratic Function In PBL Model By Using True Or False Strategy With GeoGebra-Assisted. *AlphaMath : Journal of Mathematics Education*, *6*(2), 135–142. https://doi.org/10.30595/alphamath.v6i2.8059

Antonijević, R. (2016). Cognitive Activities in Solving Mathematical Tasks: The role of a Cognitive Obstacle. *Eurasia Journal of Mathematics, Science & Technology Education*, *12*(9), 2503–2515. https://doi.org/10.12973/eurasia.2016.1306a

Asipi, L. S., Rosalina, U., & Nopiyadi, D. (2022). The Analysis of Reading Habits Using Miles and Huberman Interactive Model to Empower Students' Literacy at IPB Cirebon. *International Journal of Education and Humanities (IJEH)*, *2*(3), 117–125. https://doi.org/https://doi.org/10.58557/ijeh.v2i3.98

Azizah, N. N., Susiswo, & Sisworo. (2021). Analytical Thinking Process of Students in Solving Mathematical Problems of Quadratic Functions. *AKSIOMA: Jurnal Program Studi Pendidikan Matematika*, *10*(1), 328–338. https://doi.org/https://doi.org/10.24127/ajpm.v10i1.3440

Bauk, P., Mamoh, O., & Simarmata, J. E. (2022). Analisis Kesalahan Siswa Menggunakan Tahapan Kastolan Dalam Menyelesaikan Soal Cerita. *RANGE: Jurnal Pendidikan Matematika*, *4*(1), 28–39. https://doi.org/10.32938/jpm.v4i1.2478

Bete, H. (2019). Analisis Kesalahan Siswa SMP dalam Menyelesaikan Soal Berbentuk Higher Order Thinking (HOT) Pada Materi Aljabar. *RANGE: Jurnal Pendidikan Matematika*, *1*(1), 32–40. https://doi.org/https://doi.org/10.32938/jpm.v1i1.188

Bicer, A. (2021). Multiple Representations and Mathematical Creativity. *Thinking Skills and Creativity*, *42*, 1–17. https://doi.org/https://doi.org/10.1016/j.tsc.2021.100960

Chew, S. L., & Cerbin, W. J. (2020). The Cognitive Challenges of Effective Teaching. *Journal of Economic Education*, *52*(1), 17–40. https://doi.org/10.1080/00220485.2020.1845266

Daher, W. M., & Anabousy, A. A. (2015). Students' Recognition of Function Transformations' Themes Associated with the Algebraic Representation. *REDIMAT*, *4*(2), 179–194.

https://doi.org/10.4471/redimat.2015.1110

Díaz, V., Aravena, M., & Fores, G. (2020). Solving Problem Types Contextualized to the Quadratic Function and Error Analysis: A Case Study. *Eurasia Journal of Mathematics, Science and Technology Education*, *16*(11), 1–16. https://doi.org/https://doi.org/10.29333/ejmste/8547

Erdem, E., Zengin, Ş., & Erdem, H. (2022). Students' Ability to Make Sense of Algebraic Expressions and Their Verbal Equivalents. *European Journal of Education Studies*, *9*(1), 284–304. https://doi.org/10.46827/ejes.v9i1.4124

Gagatsis, A., Deliyianni, E., Elia, I., Panaoura, A., & Michael-Chrysanthou, P. (2016). Fostering Representational Flexibility in the Mathematical Working Space of Rational Numbers. *Bolema: Boletim de Educação Matemática*, *30*(54), 287–307. https://doi.org/http://dx.doi.org/10.1590/1980-4415v30n54a14

Hidayanti, D. R., & Hidayati, K. (2025). Identification Of Students' Errors In Solving Mathematical Literacy Problems On Quadratic Function Material. *International Journal of Progressive Sciences and Technologies (IJPSAT)*, *51*(1), 216–222. https://doi.org/10.52155/ijpsat.v51.1.7271

Hoon, T. S., Singh, P., & Halim, U. K. A. (2018). Understanding of Function and Quadratic Function among Secondary School Students in Selangor. *Asian Journal of University Education*, *14*(1), 77–88.

Islamiyah, W., Suryadi, D., & Gulvara, M. A. (2023). Student Errors on Quadratic Function Story Problems Based on The Nolting Theory. *Edumatica: Jurnal Pendidikan Matematika*, *13*(3), 191–202. https://doi.org/https://doi.org/10.22437/edumatica.v13i03.26254

Kabar, M. G. D. (2018). Secondary School Students' Conception of Quadratic Equations with One Unknown. *International Journal for Mathematics Teaching and Learning*, *19*(1), 112–129. https://doi.org/https://doi.org/10.4256/ijmtl.v19i1.94

Kartinah, Nusantara, T., Sudirman, & Daniel, T. (2020). Preliminary Study of Cognitive Obstacle on the Topic of Finite Integral Among Prospective Teacher. In E. Nofiyanto & P. M. Handayani (Eds.), *Proceedings of the 2nd International Conference on Education and Social Science Research (ICESRE 2019)* (pp. 43–46). Semarang: Atlantis Press. https://doi.org/10.2991/assehr.k.200318.008

Kartinah, Nusantara, T., Sudirman, & Daniel, T. (2021). Identifying Cognitive Obstacle on the Topic of Definite Integral among Prospective Teacher. *Ilkogretim Online - Elementary Education Online*, *20*(3), 186–192. https://doi.org/10.17051/ilkonline.2021.03.18

Koyunkaya, M. Y., & Boz-Yaman, B. (2023). Changes in Students' Mental Constructions of Function Transformations through the APOS Framework. *International Electronic Journal of Mathematics Education*, *18*(4), 1–21. https://doi.org/10.29333/iejme/13515

Ly, T., & Baba, T. (2023). Analysis of Cambodian Students' Errors in Solving Quadratic Function Problems Using Newman's Error Analysis. *Cambodian Journal of Humanities and Social Sciences*, *2*(1), 32–44.

Memnun, D. S., Aydın, B., Dinç, E., Çoban, M., & Sevindik, F. (2015). Failures and Inabilities of High School Students about Quadratic Equations and Functions. *Journal of Education and Training Studies*, *3*(6), 50–60. https://doi.org/10.11114/jets.v3i6.918

Nisa, L. C., Waluya, S. B., Kartono, & Mariani, S. (2019). Cognitive Obstacles in Interiorization of the Riemann's Sum Concept Through APOS Approach. *5th International Conference on Mathematics, Science and Education 2018 (5th ICMSE2018)*, 1–5. Bali: IOP Publishing Ltd. https://doi.org/10.1088/1742-6596/1321/2/022131

Nurrahmawati, Sa'dijah, C., Sudirman, & Muksar, M. (2021). Assessing Students' Errors in Mathematical Translation: From Symbolic to Verbal and Graphic Representations. *International Journal of Evaluation and Research in Education (IJERE)*, *10*(1), 115–125. https://doi.org/10.11591/ijere.v10i1.20819

Oktaç, A. (2022). What's New with APOS Theory? A Look Into Levels and Totality. *Avances de Investigacion En Educacion Matematica*, *21*, 9–21. https://doi.org/10.35763/aiem21.4245

Osorio, L. A. R., Bermúdez, E. A., & Jiménez, J. T. G. (2026). Encapsulation of the Defined Integral Mathematical Object of University Students: a View from APOS Theory. *Cogent Education*, *13*(1), 1–28. https://doi.org/10.1080/2331186X.2025.2588533

Rahman, T. F. binti A., & Foad, M. S. bin M. (2021). Quadratic Functions in Additional Mathematics and Mathematics: An Analysis on Students' Errors. *Academic Journal of Business and Social Sciences*, *5*(1), 1–16.

Rahmawati, A. N. L., Sa'dijah, C., & Sudirman, S. (2025). Exploring High School Students' Mathematical Representation Abilities: Translation from Verbal to Symbolic Using Higher-Order Thinking Skills. *TEM Journal*, *14*(3), 2442–2453. https://doi.org/10.18421/TEM143-48

Ramírez, R., Cañadas, M. C., & Damián, A. (2022). Structures and Representations Used by 6th Graders When Working with Quadratic Functions. *ZDM Mathematics Education*, *54*, 1393–1406. https://doi.org/https://doi.org/10.1007/s11858-022-01423-w

Ruli, R. M., Priatna, N., Prabawanto, S., & Mulyana, E. (2018). Analysis Learning Obstacle on Quadratic Function Topic. *International Journal of Information and Education Technology*, *8*(9), 681–684. https://doi.org/10.18178/ijiet.2018.8.9.1122

Safitri, R. F., Qohar, A., & Sisworo. (2025). Students' Errors in Translating Mathematical Representations from Symbolic to Graphical Form in Quadratic Functions. *AKSIOMA: Jurnal Program Studi Pendidikan Matematika*, *14*(2), 496–508. https://doi.org/https://doi.org/10.24127/ajpm.v14i2.10822

Saputra, A., Jupri, A., & Priatna, N. (2018). Various Student Strategies in Making Graph of Quadratic Functions. *International Conference on Mathematics and Science Education*, 751–755. Bandung: IOP Publishing Ltd. Retrieved from http://science.conference.upi.edu/proceeding/index.php/ICMScE/issue/view/3

Shinariko, L. J., Hartono, Y., Yusup, M., Hiltrimartin, C., & Araiku, J. (2021). Mathematical Representation Ability on Quadratic Function Through Proof Based Learning. *4th Sriwijaya University Learning and Education International Conference*, 653–659. Palembang: Atlantis Press. https://doi.org/10.2991/assehr.k.201230.177

Sholahudin, U., Oktaviyanthi, R., & Garcia, M. L. B. (2025). Analyzing Students' Cognitive Processes on Trigonometric Functions beyond 90 Degrees through the DAPIC Framework. *Range: Jurnal Pendidikan Matematika*, *7*(1), 87–109. https://doi.org/https://doi.org/10.32938/jpm.v7i1.9683

Stahl, N. A., & King, J. R. (2020). Expanding Approaches for Research: Understanding and Using Trustworthiness in Qualitative Research. *Journal of Developmental Education*, *44*(1), 26–28.

Syamsuri, S., & Marethi, I. (2018). APOS Analysis on Cognitive Process in Mathematical Proving Activities. *International Journal on Teaching and Learning Mathematics*, *1*(1), 1–12. https://doi.org/10.18860/ijtlm.v1i1.5613

Tatira, B. (2022). Undergraduate Students' Cognitive Obstacles in the Learning Power Series Concepts Using APOS. *International Journal of Social Science Research and Review*, *5*(12), 415–433. https://doi.org/10.47814/ijssrr.v5i12.820

Tendere, J., & Mutambara, L. H. N. (2020). An Analysis of Errors and Misconceptions in the Study of Quadratic Equations. *European Journal of Mathematics and Science Education*, *1*(2), 81–90. https://doi.org/10.12973/ejmse.1.2.81

Testa, I., & Catena, D. (2022). High School Students' Performances in Transitions between Different Representations of Linear Relationships in Mathematics and Physics. *Education Sciences*, *12*(11), 1–23. https://doi.org/https://doi.org/10.3390/ educsci12110776

Villamin, P., Lopez, V., Thapa, D. K., & Cleary, M. (2024). A Worked Example of Qualitative Descriptive Design: A Step-by-Step Guide for Novice and Early Career Researchers. *Journal of Advanced Nursing*, *81*(8), 5181–5195. https://doi.org/https://doi.org/10.1111/jan.16481

Wibawa, K. A. (2019). Fragmentation of the Thinking Structure of Translation in Solving Mathematical Modelling Problems. *Southeast Asian Mathematics Education Journal*, *9*(1), 25–36. https://doi.org/https://doi.org/10.46517/seamej.v9i1.71

Wilkie, K. J. (2024). Coordinating Visual and Algebraic Reasoning with Quadratic Functions. *Mathematics Education Research Journal*, *36*, 27–63. https://doi.org/https://doi.org/10.1007/s13394-022-00426-w

Yerizon, Sukestiyarno, Arnellis, Suherman, & Hevardani, K. A. (2024). Analysis of Students' Mental Construction in Understanding the Concept of Partial Derivatives Based on Action-Process-Object-Schema Theory. *Nurture*, *18*(4), 735–749. https://doi.org/10.55951/nurture.v18i4.798

Zentgraf, K., & Prediger, S. (2024). Demands and Scaffolds for Explaining the Connection of Multiple Representations: Revisiting the Bottle-Filling Task. *Journal of Mathematical Behavior*, *73*, 1–21. https://doi.org/https://doi.org/10.1016/j.jmathb.2023.101118