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Exploring the Thinking Experiences of Preservice Mathematics Teachers in Learning Geometric Transformation Proofs

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ABSTRACT

This study investigates the thinking experiences of preservice mathematics teachers in learning geometric transformation proofs through the lens of four categories of mathematical thinking: personalization, contextualization, depersonalization, and decontextualization. Employing a qualitative phenomenological approach, data were collected from 24 participants through proof tasks and semi-structured interviews. Thematic analysis identified seven key themes: dependence on examples and memorization, difficulty with formal and symbolic thinking, preference for procedural over conceptual, misconceptions of mathematical concepts, reliance on diagrammatic representations, challenges in proving surjectivity, and influence of expository teaching patterns. The findings reveal that students' thinking predominantly remained at the personalization and contextualization stages, with only limited indications of depersonalized thinking and an absence of decontextualization. These results highlight a developmental gap in abstract thinking and underscore the urgent need for instructional strategies that intentionally scaffold students' progression toward formal proof construction. This study contributes a novel theoretical perspective by applying a four-stage thinking framework to trace how preservice teachers construct and experience mathematical proofs—an approach that has not previously been employed in this domain. In addition, it offers a theoretically grounded alternative to deficit-based analyses by representing the continuum of students' thinking processes. Future research should focus on the development and implementation of didactic designs that explicitly target the transition from intuitive to formal thinking, particularly in the context of proof-based geometry instruction.

Keywords: contextualization, decontextualization, depersonalization, geometric transformation, personalization.

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Introduction

One of the critical concerns in mathematics courses for preservice mathematics teachers lies in the collective awareness that the goals of such courses should extend beyond mere mastery of mathematical concepts. These courses must also provide structured thinking experiences that support the development of professional competence in mathematics education. According to Suryadi (2019), a robust thinking experience in mathematics teacher education involves two fundamental types of thinking: (1) thinking as a mathematician, which is characterized by depersonalization and decontextualization, and (2) thinking as a mathematics teacher, marked by repersonalization and recontextualization. Interactions in the learning process that integrate these experiences serve as the foundation for preservice teachers to cultivate both disciplinary knowledge of mathematics and pedagogical capacity to teach it effectively.

These thinking experiences do not occur instantaneously but develop progressively. Preservice teachers typically begin by understanding mathematical ideas through their personal intuition and everyday contexts. These initial stages are known as personalization, where learners rely on individual impressions and subjective thinking, and contextualization, in which concepts are grasped within specific situations or representations (Kang & Kilpatrick, 1992). While these phases facilitate early engagement, they are insufficient for developing thinking that is transferable, generalized, and communicable in the formal domain of mathematics.

To support higher-level mathematical thinking, students must advance toward depersonalization and decontextualization. Depersonalization refers to the process of detaching mathematical thinking from personal experiences and formulating ideas in ways that are general, formal, and logically valid (Suryadi, 2019). Decontextualization involves removing conceptual dependence on specific situational contexts to enable abstraction, generalization, and theoretical consistency (Lerman, 2020). Together, the four stages—personalization, contextualization, depersonalization, and decontextualization—represent a continuum of cognitive development from intuitive to formal thinking. Understanding this continuum is vital not only for fostering mathematical maturity but also for designing learning trajectories that scaffold these transitions meaningfully.

One subject that can facilitate such higher-order thinking experiences—especially depersonalization and decontextualization—is the course on Geometric Transformations. Categorized as an advanced undergraduate mathematics subject, this course emphasizes formal structures, abstract formulations, and deductive proof. It systematically introduces transformations from basic relations and functions to bijective mappings over the Euclidean plane (Budiarto, 2006; Martin, 2012). Beyond its formal rigor, geometric transformations also support students' visual thinking by helping them interpret spatial relationships (Gianto et al., 2018). As Ramírez-Uclés & Ruiz-Hidalgo (2022) noted, engaging with geometric transformations enables students to justify and prove mathematical arguments while linking geometric and algebraic representations. Schenck & Nathan (2024) further emphasized that spatial thinking—closely tied to this topic—is vital for mathematical cognition and problem-solving.

While the course broadly supports these cognitive dimensions, the present study focuses specifically on the topic of transformation proofs within the course—an area that requires students to engage deeply with formal structures, abstract formulations, and deductive proof strategies. These insights underscore the course's potential to foster abstract, generalizable reasoning and make it a strategic context for cultivating depersonalized and decontextualized thinking among preservice teachers.

However, various studies have consistently reported that preservice teachers encounter significant difficulties in this subject. Kusuma & Setyaningsih (2015) highlighted challenges in grasping basic concepts and expressing transformations functionally. Noto et al. (2019) identified obstacles in visualizing geometric figures and selecting appropriate principles, leading to definitional errors. Napfiah & Sulistyorini (2021) observed persistent conceptual misunderstandings, while Indahwati (2023) noted confusion in initiating proofs and distinguishing injective versus surjective mappings. Other studies, such as those by Maifa (2019) and Aulia E et al. (2023), revealed difficulties in placing image points and thinking formally. These findings suggest that many students still rely on intuitive or procedural approaches, rather than engaging in the type of abstract thinking that this course is designed to promote.

These recurring difficulties raise an important question: Are students truly undergoing the types of structured thinking experiences this course intends to cultivate? Such evidence leads us to suspect that students' experiences of thinking in the Geometric Transformations course may not yet be ideal—particularly in terms of the expected progression from personalization and contextualization to depersonalization and decontextualization. Therefore, it becomes necessary to investigate how students actually think and reason when engaging with transformation proofs.

Despite the abundance of studies on geometric transformation learning, most research has focused primarily on identifying students' difficulties or misconceptions. For example, Kusuma & Setyaningsih (2015) and Noto et al. (2019) documented conceptual and procedural errors, while Napfiah & Sulistyorini (2021), Indahwati (2023), and Aulia E et al. (2023) highlighted recurring challenges in visualization, mapping interpretation, and formal proof construction. Their analyses generally focus on categorizing conceptual errors or procedural mistakes without investigating the cognitive trajectory behind these errors. In contrast, the present study introduces a four-category framework—personalization, contextualization, depersonalization, and decontextualization—not as a taxonomy of errors but as a developmental continuum. This lens offers a structured and theoretically grounded way to trace how students move from intuitive to formal mathematical thinking, especially in the context of proof construction—an area largely underexplored in previous studies.

This study, therefore, aims to fill that gap by exploring how preservice mathematics teachers experience the development of their thinking as they engage with proofs in geometric transformations. Specifically, it examines their thinking experiences through the lens of four progressive categories of mathematical thinking: personalization, contextualization, depersonalization, and decontextualization. Rather than evaluating proof products, the study emphasizes the lived experience of thinking. Accordingly, this study addresses the following research question: "How do preservice mathematics teachers experience the processes of personalization, contextualization, depersonalization, and decontextualization in the context of geometric transformation proofs?"

Methods

Research Design

This study adopted a qualitative phenomenological design to explore how preservice mathematics teachers constructed geometric transformation proofs. Rooted in Husserl's philosophical tradition, phenomenology emphasizes understanding experiences authentically through four stages: bracketing, intuiting, analyzing, and describing (Greening, 2019; Tuffour, 2017). This framework provided a conceptual lens for examining students' thinking across personalization, contextualization, depersonalization, and decontextualization.

The bracketing phase involved suspending researcher bias, supported by proof tasks designed to elicit both intuitive and formal responses. Written work served as initial data, followed by semi-structured interviews during the intuiting phase to capture students' reflective reasoning. Interviews were recorded, transcribed, and examined empathically. In the analyzing phase, responses were coded and categorized according to the four thinking types. Finally, the describing phase synthesized patterns into a narrative, identifying dominant thinking stages and how they emerged through specific themes and codes.

Participants

This study involved 24 preservice mathematics teachers from the Mathematics Education program at Universitas Timor, Indonesia. Participants were selected through criterion sampling, a purposive technique based on predefined criteria (Tuffour, 2017), to ensure relevant experience and meaningful contributions to the phenomenon under investigation (Alase, 2017). Inclusion criteria included enrollment in the Geometric Transformations course, completion of prerequisite coursework, demonstrated proficiency in basic proof techniques (verified through academic records and instructor confirmation), and voluntary participation in both tasks and interviews.

Instrument and Data Collection Procedures

Data collection employed two instruments: proof tasks and semi-structured interviews. The tasks included definition-based and formula-based items designed to elicit students' thinking. Follow-up interviews probed participants' thinking during proof construction.

Data Analysis

In line with the phenomenological approach, the data analysis followed two main stages after the bracketing and intuiting phases—namely analyzing and describing. These stages aimed to uncover the structure of participants' thinking as they constructed transformation proofs and to examine how their thinking aligned with one or more of the four thinking categories.

The analysis involved three key steps: (A) Coding (Analyzing), where data from written responses and interviews were examined to identify thinking patterns and signs of specific stages of thinking; (B) Categorization (Analyzing), in which the resulting codes were grouped into broader categories representing dominant thinking types—personalization, contextualization, depersonalization, or decontextualization—based on general indicators developed from each category's definition; and (C) Interpretation and Presentation (Describing), where the findings were interpreted to determine dominant thinking patterns among participants.

Table 1. *Indicators of Thinking Categories*

Thinking Category	Indicator
Personalization	Students rely on personal intuition, such as memorized examples or informal strategies based on personal experience.
Contextualization	Students use visual illustrations, arrow diagrams, or relational forms that are familiar to explain mathematical ideas.
Depersonalization	Students apply formal definitions in proving geometric transformations, including formally demonstrating injectivity and surjectivity.
Decontextualization	Students present general formal arguments that are not dependent on visual contexts or function diagrams.

Results and Discussion

Research Result

A. Bracketing - Filtering Assumptions and Designing Tasks

The tasks were specifically designed to elicit thinking experience across this continuum. These tasks were categorized into two types:

1. Definition-Based Transformation Task

   Let point M, line segment $\overline{AB}$ and line s be given, such that $\overline{AB}$ is parallel to *s,* and the distance from M to line s is twice the distance from M to segment $\overline{AB}$. Define a Function h from $\overline{AB}$ to s that maps any point $P \in \overline{AB}$ to the point $P' = \overleftrightarrow{MP} \cap s$. Prove whether or not the functions h is a transformation.
2. Formula-Based Transformation Task

   Determine whether the function $F(x,y) = (2x+1, y-x)$ is a transformation.

A total of 24 preservice teachers participated in this written task. The results of participants' written responses are summarized in the following table:

Table 2. *Summary of Participants' Responses to the Tasks*

Task	Category	Number of Participants
1 (Definition-Based)	Correct	0
	partially correct	3
	Incorrect	10
	No Respon	11
2 (Formula-Based)	Correct	0
	partially correct	7
	Incorrect	14
	No Respon	3

Responses were coded as "partially correct" if participants demonstrated conceptual direction but failed to complete the logical thinking or use formal terminology accurately.

B. Intuiting

The analysis of participants' written responses and follow-up interviews provided initial insights into their thinking processes as they engaged with the two proof tasks: (1) a definition-based transformation task and (2) a formula-based transformation task. This section begins with findings from the first task, which required participants to construct a transformation based on a geometric definition involving points, lines, and distances.

Participant's drawing depicted a point labeled M (see figure 1), a function $f(h)$ and a $\overline{AB}$, yet it lacked clarity regarding the spatial relationships and transformation properties described in the task. To uncover the thinking behind this attempt, the researcher conducted a follow-up interview. When asked to explain the components of the diagram, participant simply referred to point M as "a point" and line *s* as "a line", offering no elaboration on their respective roles or relationships in the transformation setup.

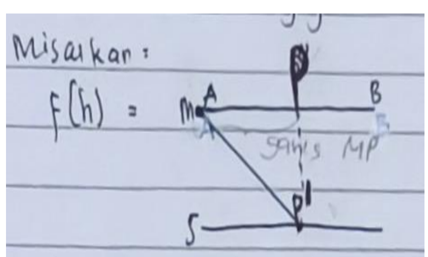

Figure 1. *Participant's Visualization for task 1*

The researcher then explored participant's grasp of the image set of a function. When asked about the range of the transformation, participant responded, *"The range is P'"*. Upon clarification—*"Is P' the only point in the range? Or are there other points as well?"*— participant reaffirmed, *"In this question,* *the range is only P'."* This indicates that participant perceived the range as a single point, not a set of image points under transformation. Such thinking reflects a narrow interpretation of the function concept and suggests difficulty in recognizing generalization within transformation mappings.

In response to a question about why he did not attempt to prove the injectivity and surjectivity of the transformation, the participant stated he was unsure how to construct a proof. This is notable considering participant had already created a visual model and identified some relational components. Participant explained that he had not yet understood how to use contradiction—a standard method in proving injectivity—and expressed uncertainty about how to approach proofs of surjectivity as well. These difficulties were further confirmed when the researcher asked to see their class notes on proof strategies, revealing gaps in both content knowledge and thinking readiness.

In the follow-up session, the researcher asked participant to explain why the concepts of injectivity and surjectivity were difficult to understand. He admitted that initiating a proof was challenging and that even previously covered material remained difficult to grasp. To gain further insight, the researcher asked him to refer to their class notes on proof strategies. As shown in figure 2, the notes were incomplete, particularly on topics related to injective and surjective functions.

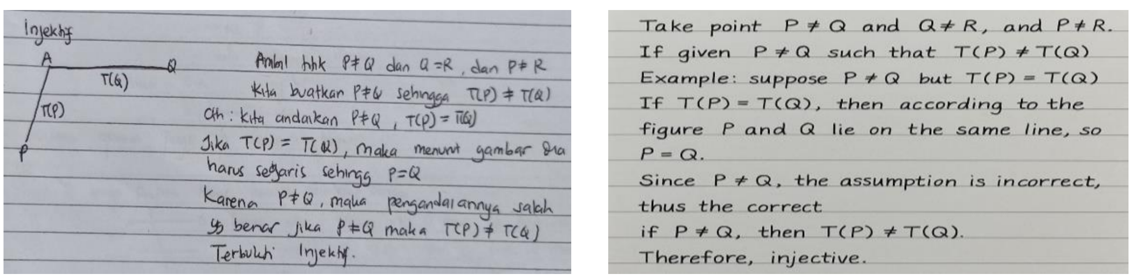

Figure 2. *Participant's Class Notes*

During the interview, the researcher inquired which part of the injectivity proof participant found confusing. Participant responded that they did not understand proof by contradiction and were unsure why points P and Q were used. When asked whether they understood the assumption $P \neq Q$ and the reasoning behind choosing arbitrary points at the beginning of the proof, participant acknowledged his confusion.

The researcher then explored participant's understanding of the term *assumption*. He stated that an assumption is used to test whether the assumption leads to a contradiction —that is, whether it results in something being both true and false. However, when asked to elaborate on the meaning of contradiction, participant simply explained that it means "something equals something else, and at the same time it doesn't." For example, if $P = Q$ then $T(P) \neq T(Q)$. When asked to identify the contradiction in this context, he replied: "The part where they're not equal, $T(P) \neq T(Q)$"

This exchange revealed that the participant experienced significant difficulty in understanding each step of the injectivity proof. The confusion originated with the initial assumption, as the participant did not grasp the rationale for making such an assumption or the meaning of contradiction within the proof structure. This was evident in the student's explanation, where contradiction was reduced to simply "not equal," without a full understanding of its logical role in the argument.

The next item addressed in the study was the formula-based transformation task, which explored the ability of participants to evaluate a function and determine whether it qualified as a transformation. The results revealed a consistent pattern of difficulty. Of the 24 participants, only 7 produced partially correct responses, 14 gave incorrect answers, and 3 did not attempt the problem at all.

Among those who answered incorrectly, many adopted a similar strategy: they attempted to evaluate the function by selecting specific example points and substituting them into the given expression $F(x, y)$. Rather than engaging with the underlying properties of the transformation, these participants focused on generating a list of input-output pairs.

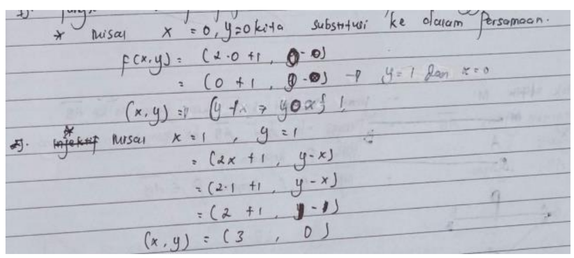

Figure 3. *Participant's Response to Problem 2*

As seen in figure 3, participant began by choosing the point $(x, y) = (0,0)$ and substituting it into the function formula. He proceeded to compute corresponding image points using basic substitution, but did not attempt a formal justification or consider the definition of transformation. To further understand the rationale behind this approach, the researcher conducted a follow-up interview.

The participant was only able to provide this brief response, explaining that the substitution of values was intended "to determine the points x and y." Further inquiry revealed that this approach stemmed from a direct replication of an example found in the instructional material, as shown in figure 4. This suggests that the student relied heavily on previously encountered solutions, even when the new problem was only a minor variation of the example. When asked why they did not proceed to prove injectivity and surjectivity, the participant responded that the differences made the task too difficult to attempt.

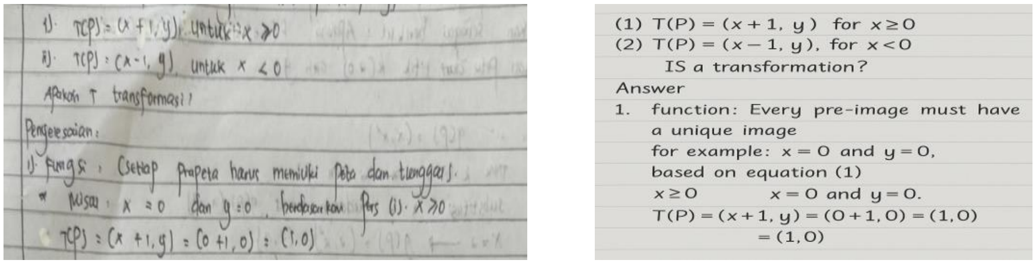

Figure 4. *Participant's Class Notes*

Such reliance on memorized procedures also manifested in the use of diagrammatic representations. As illustrated in figure 5, some participants attempted to demonstrate injectivity using an arrow diagram—a format commonly used in secondary-level textbooks. This tendency reflects contextualized thinking, where students rely on familiar visual forms rather than abstract mathematical definitions. While these visualizations helped illustrate mappings, participants often struggled to move beyond them to engage in symbolic or formal proof processes. Even among those who provided partially correct responses, many failed to complete the logical argument fully. They typically addressed the injectivity component alone, while the surjectivity proof was either omitted or approached superficially.

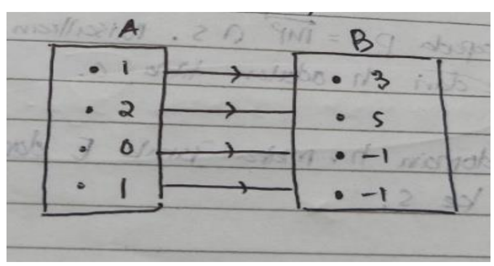

Figure 5. *Arrow Diagram Representation of the Function*

A follow-up interview was conducted to examine contextual factors influencing participants' thinking. There are three instructional patterns emerged: (1) Lectures predominantly used expository methods, emphasizing example-following over exploration; (2) Students were rarely engaged in constructing arguments independently, limiting opportunities for depersonalized or decontextualized thinking; (3) When solving new tasks, participants relied on replicating previously taught examples rather than forming original reasoning. These patterns suggest that the dominance of early-stage thinking may be rooted in how proof-based content is delivered and reinforced in the classroom.

C. Analyzing

Following the initial exploration of participants' responses through the intuiting phase, the next stage involved a systematic analysis to identify patterns of thinking. The data from both the written tasks and follow-up interviews were coded to extract recurring features of participants' thought processes. Thematic analysis revealed seven recurring patterns that characterized participants' thinking during transformation proof activities.

Table 3. *Emergent Thinking Themes in Geometric Transformation Proof Tasks*

Theme	Description
Dependence on Examples and Memorization	Most participants relied on examples previously encountered in their notes or classroom instruction. When presented with a problem whose format slightly differed from the examples they had memorized, they struggled to transfer their knowledge.
Difficulties with Formal and Symbolic Thinking	Participants exhibited limited ability to use symbolic representations accurately. Many responses lacked formal structure, and several participants expressed confusion about the logical flow required in constructing proofs.
Preference for Procedural over Conceptual	Rather than constructing logically justified arguments, many students engaged with transformation tasks by following procedural routines—such as plugging in values or applying formulaic steps.
Misconceptions of Mathematical Concepts	Students frequently misunderstood key geometric and logical ideas. For example, some interpreted the concept of intersection as overlap rather than a defined shared point, and others misunderstood what symbols like P′ represented in function mappings.
Use of Diagrammatic Representations	Several participants employed visual tools—especially arrow diagrams—to support their thinking. However, they often failed to translate these visualizations into formal arguments or generalizable structures.
Challenges in Proving Surjectivity	While a few participants were able to demonstrate injectivity, they typically did not proceed to proving surjectivity. Some explicitly stated that they found it more difficult, even when visual evidence of mapping was present.
Influence of Expository Teaching Patterns	Students frequently modeled their responses on examples previously provided by instructors. Interviews revealed that classroom instruction was often centered on worked examples, which students imitated without developing independent thinking.

After identifying seven dominant thinking patterns, each theme was analyzed to determine its alignment with the four categories of thinking outlined in Table 1. The first theme—dependence on examples and memorization—reflects personalization, as participants relied on recalled classroom examples and informal strategies rather than formal thinking. The second theme, difficulties with formal and symbolic thinking, indicates a lack of depersonalization, evidenced by students' limited use of formal notation and inability to construct logically coherent proofs. The third and fourth themes—

preference for procedural approaches and misconceptions of fundamental concepts—straddle personalization and contextualization. These themes reveal students' reliance on algorithmic steps or surface-level representations, such as diagrams or intuitive interpretations of symbols, rather than conceptual generalizations.

The fifth theme, reliance on diagrammatic representations, aligns strongly with contextualization. Although visual tools like arrow diagrams were frequently used, students struggled to abstract their reasoning beyond these aids. The sixth theme, challenges in proving surjectivity, underscores the limited emergence of depersonalized thinking, as students often failed to generalize or formalize their visual insights. Finally, the seventh theme—dominance of expository teaching patterns—reinforces personalization, with participants imitating pre-modeled proofs rather than constructing arguments independently. Overall, the categorization shows that participants' thinking was largely confined to the personalization and contextualization stages, with only partial signs of depersonalization and no clear evidence of decontextualized thinking.

Table 4. *Mapping of Emergent Themes to Thinking Categories*

Emergent Theme	Thinking Category
Dependence on Examples and Memorization	Personalization
Difficulties with Formal and Symbolic Thinking	Depersonalization (lacking)
Preference for Procedural over Conceptual	Personalization, Contextualization
Misconceptions of Mathematical Concepts	Personalization, Contextualization
Use of Diagrammatic Representations	Contextualization
Challenges in Proving Surjectivity	Depersonalization (limited)
Influence of Expository Teaching Patterns	Personalization

Despite the dominance of thinking rooted in personalization and contextualization, the data revealed limited signs of emerging depersonalized thinking. Some participants began engaging with formal definitions, especially those related to injectivity and surjectivity, though their attempts were often incomplete or flawed. These early efforts, however, indicate a shift toward deductive thinking and abstract principles. Rather than viewing them as deficiencies, such responses suggest developmental progress and a readiness to move beyond surface-level understanding—especially when supported by targeted instructional scaffolding. Framing these moments as growth points aligns with a strengths-based perspective and informs future pedagogical strategies.

Discussion

The final stage of this phenomenological investigation—describing—aims to synthesize the key findings from the analyzing phase in order to address the research question holistically. This phase goes beyond merely presenting participant responses; it involves interpreting how each emergent theme reflects the participants' thinking engagement with geometric transformation proofs. Through this interpretive lens, the study traces the progression and challenges in students' thinking across the four categories of thinking: personalization, contextualization, depersonalization, and decontextualization. By connecting these findings to the developmental continuum of thinking, this section provides a comprehensive understanding of how preservice mathematics teachers experience and navigate the process of constructing mathematical proofs, thereby fulfilling the study's objective.

The findings suggest that the thinking experiences of preservice teachers were largely situated in the stages of personalization and contextualization. Students predominantly relied on examples, classroom routines, and informal thinking to engage with transformation tasks. This tendency is consistent with the work of Kusuma & Setyaningsih (2015), who observed that students often imitate familiar problem structures rather than reason flexibly across contexts. Similarly, Lerman (2020) noted that personalization reflects thinking grounded in intuition and subjective familiarity, which aligns with the observed patterns of memorized examples and mechanical substitution during problem solving.

In addition, contextualization was evident when participants used visual aids such as diagrams or related the tasks to classroom explanations. While these strategies helped scaffold their initial understanding, they also became cognitive anchors that restricted abstraction. This aligns with findings by Maifa (2019), who reported that students had difficulty abstracting beyond visual forms when learning about mappings and geometric relations. The persistence of contextualized thinking, as also emphasized by Napfiah & Sulistyorini (2021), limited students' capacity to reason beyond surface representations.

Signs of depersonalization, while limited, did emerge among a subset of participants. These individuals attempted to invoke formal definitions—particularly of injectivity and surjectivity—during proof construction. Although their justifications were frequently incomplete, such efforts indicate a shift from intuitive to more structured mathematical thinking. Houston (2009) and (Suryadi, 2019) both emphasized that the development of depersonalized thinking is crucial for students to construct logically valid proofs independently. The current findings extend this by showing that even minimal engagement with formal structures represents meaningful cognitive progress that should be supported in teacher education.

Decontextualized thinking —where students operate beyond specific representations or task formats—was rarely observed. A few participants expressed the need to generalize their understanding or question whether examples seen in class applied to unfamiliar cases. However, these reflections did not translate into consistent abstraction or application. Lerman (2020) describes decontextualization as the hallmark of advanced mathematical thinking; its absence in this study reflects the need for more deliberate instructional design aimed at cultivating this level of thinking.

Furthermore, several misconceptions regarding basic mathematical concepts—such as line intersection, distance, or mapping notation—revealed the fragility of participants' conceptual understanding. These errors are indicative of thinking that remains entangled in personalization or contextualization, without firm grounding in structural principles. Studies by Indahwati (2023) and Maifa (2019) confirm that preservice teachers often struggle to distinguish between conceptual elements such as domain, codomain, or image points, suggesting a systemic gap in developing abstract thinking in geometry courses.

Despite these challenges, the study also found encouraging signs of cognitive growth. Some participants demonstrated the ability to move beyond rote procedures and attempted to reconstruct arguments using definitions. Others showed a readiness to generalize or reflect on the limitations of example-based learning. These tendencies align with Suryadi (2019) assertion that thinking as a mathematician begins when learners detach from personal and situational thinking. Although such shifts were emergent and limited, they signal a developmental trajectory that could be strengthened through pedagogical interventions. The reproduction-oriented instructional model described by participants—centered on expository teaching and repetition of worked examples—may have contributed to their cognitive stagnation. Anwar et al. (2021) and Sundawan et al. (2018) similarly found that students exposed to overly directive teaching struggled to construct independent arguments. The findings of this study extend that critique by showing how such approaches reinforce early-stage thinking and inhibit progression toward formal mathematical thinking.

In direct response to the research question—*"How do preservice mathematics teachers experience the processes of personalization, contextualization, depersonalization, and decontextualization in the context of geometric transformation proofs?"*—this study found that participants predominantly experienced thinking within the stages of personalization and contextualization. These stages were characterized by reliance on memorized examples, procedural routines, and surface-level representations. While attempts at depersonalized thinking were observed—such as using formal definitions or initiating proof structures—these efforts were often incomplete and lacked logical coherence. Decontextualized thinking, where students abstract from specific visual or situational cues to engage with general mathematical structures, was notably rare. Overall, the findings reveal that preservice teachers' experiences with transformation proofs remain largely rooted in intuitive and contextual understandings, with only limited and emergent transitions toward abstract and formal thinking.

Conclusion

This study offers a nuanced synthesis of preservice mathematics teachers' thinking as they engaged with geometric transformation proofs through the four-stage thinking: personalization, contextualization, depersonalization, and decontextualization. The analysis of seven emergent thinking themes demonstrates a clear dominance of personalization and contextualization, where students relied heavily on memorized examples, visual aids, and procedural routines. Attempts at depersonalization—though limited—emerged in students' use of formal definitions and efforts to construct deductive arguments, while decontextualized thinking was virtually absent. Theoretically, the study contributes a structured lens for examining thinking development, offering an alternative to deficit-based analyses that focus solely on errors or misconceptions. Practically, it provides insights for designing a didactic framework that fosters depersonalized and decontextualized thinking. However, the study is limited by its context-specific sample and reliance on task-based interviews, which may restrict generalizability. Future research should investigate how sustained classroom interactions, collaborative proof construction, and instructional scaffolds influence the trajectory of thinking across diverse mathematical topics.

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