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Analyzing Students' Cognitive Processes on Trigonometric Functions beyond 90 Degrees through the DAPIC Framework

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ABSTRACT

Understanding trigonometric functions at angles beyond 90 degrees presents unique cognitive challenges for students, requiring the integration of conceptual, procedural, and representational knowledge. However, research exploring how students cognitively process such problems, especially within a structured framework, remains limited. This study aims to analyze students' cognitive processes in solving trigonometric problems involving beyond-90-degree angles through the DAPIC framework (Define, Assess, Plan, Implement, Communicate), offering a novel application of DAPIC to this underexplored context. A qualitative case study approach was employed, involving six 11th-grade students from a public high school with varying cognitive levels. Data were collected through diagnostic tasks, think-aloud protocols, and semi-structured interviews, and were analyzed by mapping students' thinking patterns, defined as the recurring sequences of cognitive moves and representation use observed across DAPIC stages. The results reveal that high-performing students demonstrated flexible shifts between symbolic, graphical, and unit circle representations and were capable of self-regulating their errors reflectively. In contrast, students with moderate and low performance encountered difficulties in identifying the quadrant of angles and understanding the periodic nature of trigonometric functions, particularly during the Assess and Plan stages. Adaptive strategies, such as visual re-checking or intuitive quadrant reasoning, emerged spontaneously but were not always effective. These findings suggest that DAPIC serves as a systematic tool for capturing the dynamics of students' thinking processes and offers valuable insights for developing deeper instructional strategies in trigonometry, especially for topics involving beyond-90-degree angles.

Keywords: Adaptive strategies, cognitive process, mathematical representation, trigonometric functions, qualitative case study.

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Introduction

Trigonometry is a fundamental topic in the secondary school mathematics curriculum, with strong relevance to advanced fields such as calculus, physics, engineering, and information technology (Bekene Bedada & Machaba, 2022; Cirneanu & Moldoveanu, 2024). Understanding trigonometric functions is not only essential for solving academic problems but also for developing analytical thinking and addressing real-world phenomena involving periodicity and rotation, such as oscillatory motion, signal processing, and mathematical modelling in programming (Haigh, 2019; Hidayat et al., 2023). This demands students' proficiency in interpreting mathematical representations across various forms, graphical, symbolic, verbal, and visual which (Ngu & Phan, 2023; Sholahudin & Oktaviyanthi, 2025), serve as a foundation for building deep conceptual understanding (Obeng et al., 2024). Within the framework of the *Merdeka Curriculum* and international assessments like PISA, such abilities are classified under abstract reasoning, logical thinking, and generalization skills (Ndari et al., 2023; Popkewitz, 2022).

However, based on preliminary classroom observations conducted during pilot implementation in one public high school**,** many students encounter significant difficulties in understanding trigonometric functions, particularly when dealing with angles greater than 90 degrees, such as 135°, 210°, or even 450°. These challenges are not limited to procedural mistakes but often involve deeper conceptual misunderstandings. For instance, students frequently struggle to identify the correct quadrant of a given angle, assign the appropriate sign to trigonometric values based on the quadrant, or accurately utilize the unit circle. A common error is stating that $\sin(210°) = \sin(30°)$, neglecting the negative sign due to the angle's position in the third quadrant. These examples illustrate that students' comprehension of the periodicity, symmetry, and representational nature of trigonometric functions remains incomplete (Modabbernia et al., 2023).

Previous research has categorized various student errors in learning trigonometric concepts (Nordlander, 2022; Obeng et al., 2024; Sekgoma & Salani, 2023). Nevertheless, many of these studies adopt a descriptive approach focused mainly on final outcomes or classifications of mistakes. There remains a notable gap in exploring how students' thinking evolves during the problem-solving process. Process-oriented approaches are needed to uncover the cognitive dynamics that underlie students' conceptual development and strategy formulation. This study addresses that gap by adopting a process-oriented approach grounded in the DAPIC framework (Define, Assess, Plan, Implement, Communicate), applied to problem-solving tasks involving trigonometric functions with angles greater than 90 degrees.

In contrast to previous studies that emphasize results, this study reconstructs students' thinking paths step-by-step. The DAPIC framework offers a structured lens through which to observe how students define the problem, assess relevant information, devise a plan, implement the strategy, and communicate their understanding (İncelenmesi et al., 2024; Meier et al., 1996; Santos-Trigo, 2020). To capture students' thought processes in real time, the study employs think-aloud protocols and semi-structured interviews, with particular attention to their use of mathematical representations throughout.

Accordingly, this study focuses on analyzing students' thinking patterns in solving trigonometric function problems involving angles greater than 90 degrees, using the DAPIC framework as a guiding lens. The novelty of this research lies in two aspects: (1) the use of a process-oriented approach to trace students' reasoning across contextual stages, and (2) the application of the DAPIC framework to the domain of trigonometry, a topic rarely examined through this lens in prior studies. The findings are expected to contribute to the design of mathematics instruction that is more adaptive and responsive to students' conceptual challenges. Specifically, the study aims to: (1) identify thinking patterns exhibited by students in solving trigonometric problems with angles exceeding 90°; (2) analyze the forms of mathematical representation employed at each stage of their thinking; and (3) describe the types of conceptual errors and adaptive strategies that emerge during the problem-solving process.

Methods

This study employed an embedded multiple-case study approach (Hunziker & Blankenagel, 2024), which involves the examination of multiple cases (students) within a single bounded system (problem-solving task) while allowing for both within-case and cross-case analysis (Moore et al., 2024). This approach enables a contextualized understanding of students' cognitive processes and how they manifest differently depending on proficiency levels and representational fluency (Carron et al., 2023; Gillen et al., 2021). The primary focus was placed on each subject's cognitive processes rather than solely on their final answers, making this design appropriate for investigating the depth of students' conceptual understanding and their use of representational strategies (Adler et al., 2025; Hegedus & Otálora, 2023).

Participants in this study were six 11th-grade students from a public senior high school in Serang, Banten who had previously studied trigonometry. The subjects were selected purposively based on their pretest results to represent a range of cognitive levels (high, medium, and low). The pretest was a mathematics-focused diagnostic test consisting of five non-routine trigonometric problems covering concepts such as angle reduction, quadrant identification, periodicity, and use of the unit circle. This classification was guided by the students' pretest scores, with those scoring above 80% considered high, 50%–79% as medium, and below 50% as low (Newton & Martin, 2013). In addition to score-based criteria, students' willingness to engage in think-aloud protocols and follow-up interviews was also taken into account during the selection process. All participants obtained written consent from their parents and the school administration prior to participating in the study.

The primary instruments used in the study included:

1. Trigonometric Function Problem Sheet

This sheet contained six problem-solving items focusing on angles greater than 90°, including 135°, 210°, 270°, and 450°. The items were designed to probe students' understanding of quadrant concepts, periodic properties, and the use of the unit circle representation. Each item required students to express their solutions in symbolic form, graphical representation, and verbal explanation, enabling cross-representational analysis. As shown in Table 1, these tasks served as the foundation for eliciting students' thinking patterns and were used during the think-aloud sessions.

Table 1. *Trigonometric Function Problem Sheet*

No.	Problem	Competency Targeted
1	"Determine the value of $\cos(1080°)$. Can you simplify the angle first? Explain your method."	Tests students' awareness that 1080° can be reduced to an equivalent angle within 0°–360° due to cosine periodicity
2	"Is 450° a special angle? How would you find the value of $\cos(450°)$?"	Encourages students to identify angles greater than 360° and their classification as "special angles"
3	"1020° is a large angle. What should be done before finding $\sin(1020°)$?"	Targets students' awareness of angle reduction and understanding that 1020° exceeds two full rotations
4	"Do you consider 570° a special angle? Explain."	Elicits understanding of reference angles and the distinction between large and "special" angles
5	"To find $\tan(1170°)$, what do you think is the first step?"	Assesses recognition of fundamental steps such as subtracting multiples of 360°
6	"$\cos(990°)$ = ... Before answering, do you realize this angle can be simplified? How?"	Encourages immediate recognition that 990° is equivalent to 270°, testing intuitive understanding

2. Think-Aloud Protocol Guide

This guide was used to prompt students to verbalize their thought processes while solving the problems. Think-aloud protocols are a method to capture cognitive activity in real time by having participants articulate their thoughts while performing a task (Pratt & Hodges, 2023). It included example instructions such as "Tell me what you're thinking as you read this question," ensuring consistency in eliciting responses across participants. Refer to Table 2 for the complete think-aloud guidance framework used during data collection.

Table 2. *Think-Aloud Protocol Guide*

DAPIC Stage	Cognitive Process Indicator	Guiding Question
Define	Ability to identify the type and meaning of the angle	"What kind of angle is referred to in this problem?"
Assess	Identifying key information (quadrant, reduction, function)	"What information do you need to solve this problem?"
Plan	Formulating strategies and considering alternatives	"What is your first step and why did you choose it?"
Implement	Executing strategy and explaining the process	"What are you doing now? Why?"
Communicate	Reflecting on and justifying the answer	"What is your final answer? Are you confident? Explain why."

3. Semi-Structured Interview Guide

Semi-structured interviews, developed based on the stages of the DAPIC framework, were conducted after the think-aloud process to triangulate data and gain deeper insights into students' reasoning (Brown & Danaher, 2019). The interview questions were designed to explore the rationale behind students' strategy selection, changes in their approach, and their conceptual understanding of trigonometry. As shown in Table 3, the questions were aligned with each stage of the DAPIC framework to ensure comprehensive cognitive coverage and strengthen the interpretation of students' thinking processes.

Table 3. *Semi-Structured Interview Guide*

DAPIC Stage	Interview Indicator	Probing Question
Define	Understanding of large angles and the need for reduction	"Why did you feel it necessary to reduce the angle?"
Assess	Selecting relevant information or comparing representations	"What did you immediately notice about the angle?"
Plan	Reasoning behind choosing a specific strategy	"Why did you choose that method over others?"
Implement	Awareness during strategy execution and recognizing errors	"Did anything cause you to hesitate while solving?"
Communicate	Strength of argument and clarity in presenting results	"How would you convince others that your answer is correct?"

4. Researcher Observation Sheet

The researcher observation sheet was used to record students' non-verbal expressions, thinking pauses, and their interactions with visual representations (e.g., drawing the unit circle or visualizing graphs), thereby helping to validate interpretations derived from the think-aloud data (Cohen et al., 2017). Refer to Table 4 for the structure of the observation sheet employed during task implementation.

Table 4. *Researcher Observation Sheet*

DAPIC Stage	Observed Aspect	Sample Note
Define	Signs of confusion or hesitation while reading the question	"Frowned, paused for 7 seconds after reading 1170°"
Assess	Pointing to graphs or sketching the unit circle	"Drew auxiliary line in quadrant IV while speaking softly"
Plan	Behavioural changes when switching strategies	"Erased solution, said: 'Let me try another way'"
Implement	Fluency in writing and verbalizing steps	"Wrote quickly, explained while pointing to numbers"
Communicate	Body language during explanation	"Smiled, confident tone while stating $\cos(0°) = 1$"

5. Audio-Video Recordings

Audio-video recordings were employed to ensure transcription accuracy and to facilitate detailed behavioural analysis, particularly regarding gesture, hesitation, and error correction (Parameswaran et al., 2020). All think-aloud sessions and interviews were recorded, with audio data used to confirm verbal accuracy and video recordings capturing students' manipulative actions involving diagrams or symbolic representations. As shown in Table 5, the video coding framework categorized these actions by representational type, duration, and cognitive function to support deeper interpretative analysis.

Table 5. *Audio-Video Recordings*

DAPIC Stage	Visual/Audio Element	Video Coding Purpose
Define	Pause duration when reading the problem	Indicator of initial cognitive load
Assess	Verbal statement "needs to be reduced..."	Reveals activation of periodicity concept
Plan	Motion of drawing graphs or rotating paper	Visualization of internal strategy
Implement	Self-correction with "oops"	Indicator of metacognitive control
Communicate	Raised voice during explanation	Indicator of confidence and concept mastery

This study employed a structured qualitative procedure involving participant selection, cognitive task execution, and data analysis. It began with a pretest to identify students' conceptual understanding. Based on the results, six participants (S1–S6) were selected to represent diverse cognitive profiles. If a candidate did not meet the inclusion criteria (e.g., willingness to participate, availability, and score classification), another student was selected based on the same criteria. This selection method aligns with purposive sampling principles in qualitative research (Cash et al., 2022; Robinson, 2023). Selected participants completed a problem-solving task using the think-aloud protocol, verbalizing their thought processes while working on visual and symbolic mathematical problems. Minimal prompting was provided to maintain natural responses. If the think-aloud data were incomplete or invalid, the session was repeated or the participant was replaced. Valid sessions were followed by semi-structured interviews to explore students' reasoning, use of representations, and conceptual strategies. Data from think-aloud sessions, interviews, and observations were then analyzed using the DAPIC framework (Define, Assess, Plan, Implement, Communicate). Inter-rater consistency was checked; if disagreements occurred, the team conducted discussions and re-analysis. Finally, triangulation was performed by comparing all data sources to ensure consistency and validate findings, leading to a comprehensive understanding of students' problem-solving and visual reasoning processes. Here is the flowchart summarizing the procedure described above in Figure 1.

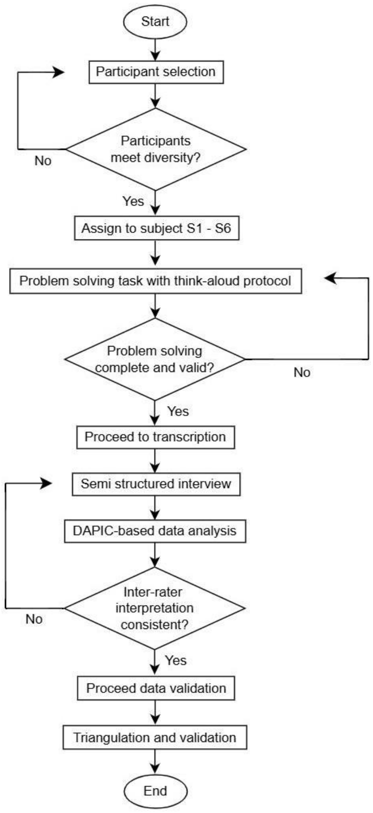


Figure 1. *Research Procedure Flowchart based on the DAPIC Analysis Framework*

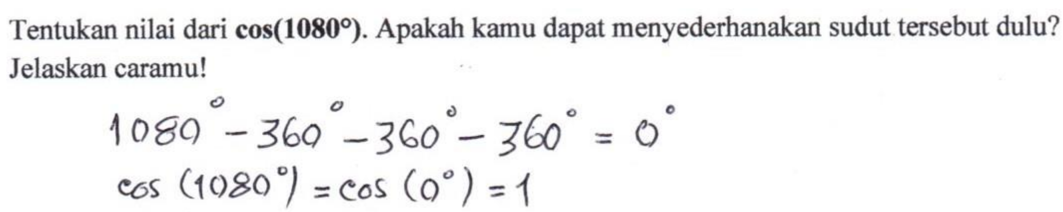
Data were analyzed qualitatively through five DAPIC-patterned stages: Define, Assess, Plan, Implement, and Communicate (Meier et al., 1996; Santos-Trigo, 2020). Transcripts from think-aloud protocols and interviews were examined to identify strategic patterns, types of representations, conceptual errors, and transitions in understanding across stages (Li et al., 2024; Pratt & Hodges, 2023). Open coding was conducted to categorize student responses, which were then triangulated across verbal data, written work, and interview outcomes (Craig et al., 2021). In this study, the term "thinking pattern" refers to the recurring sequence of cognitive moves and representation use observed during problem-solving, as structured by the DAPIC stages (Kholid et al., 2022). These were traced through verbal protocols, behavioural indicators, and written responses, and then coded into cognitive categories such as symbolic reasoning, graphical interpretation, strategic planning, and metacognitive reflection (Craig et al., 2021; Liu et al., 2023). Triangulation was achieved through methodological triangulation (e.g., think-aloud, interviews, observations) and data triangulation (multiple participants and instruments). Additional validity checks included member checking, where participants reviewed interpretations of their responses, and peer debriefing, involving discussions with fellow mathematics education researchers to reduce bias (Almutairi & Shraid, 2021; Planas-Lladó et al., 2021).

Results and Discussion

These results were informed not only by students' written responses but also by triangulated qualitative data sources. Semi-structured interviews provided insight into students' rationale behind strategy choices and error reflection. Observation sheets captured non-verbal behaviors such as hesitation, diagram construction, or gestural referencing during problem-solving. Audio-video recordings enhanced the analysis by enabling the identification of pauses, visual cues, and clarification of spoken reasoning.

In this study, while the DAPIC framework (Define, Assess, Plan, Implement, Communicate) was used as a structural guide for analyzing the sequence of students' responses, each stage was also interpreted through the lens of *thinking patterns*. These patterns refer to the cognitive tendencies, strategies, and representational preferences that students employed when solving the trigonometric problems. Therefore, the term *thinking patterns* is consistently used throughout this section to emphasize not just the procedural sequence of problem-solving, but the underlying cognitive approaches that emerged within and across DAPIC stages.

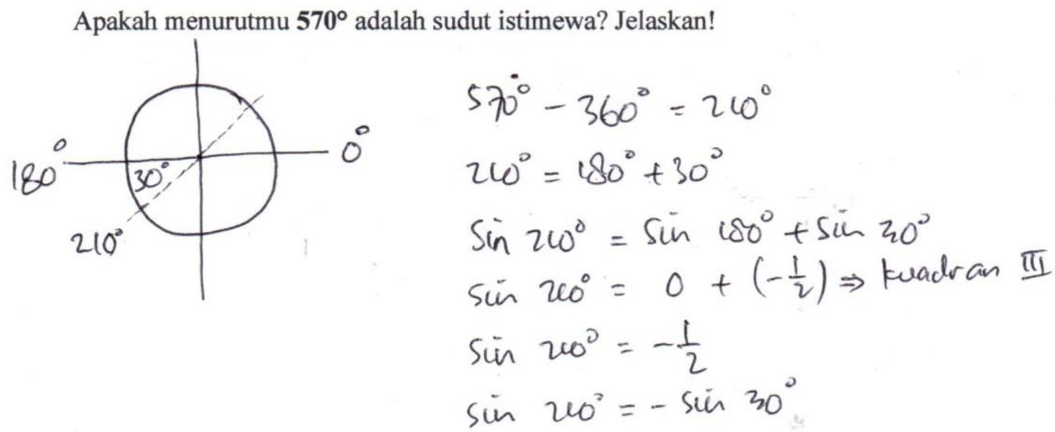
Subjects S1 and S6 – High Cognitive Level



The translated problem:
Determine the value of $cos(1080°)$. *Can you simplify the angle first? Explain your method!*

Figure 2. *Responses of Subject S1*

Based on Figure 2, Subject S1 demonstrated a tendency towards efficient and numerically-oriented mathematical thinking in solving trigonometric problems with angles exceeding 360°. When presented with the problem $\cos(1080°)$, S1 promptly performed successive subtractions of 360° until reaching the equivalent angle of 0°. He directly stated that $\cos(1080°) = \cos(0°) = 1$. This step reflects an understanding of the periodicity of the cosine function, executed without reliance on visual aids or representations such as the unit circle. During observation, S1 appeared confident and calm, completing the problem swiftly without hesitation or confusion. A think-aloud quote, *"I just subtracted three times 360 to get zero degrees. Since* $cos(0°)$ *is one, it must be correct,"* indicated that S1 relied purely on numerical computation and memorization of trigonometric values at special angles, without engaging in further exploratory or reflective processes. This characteristic reveals a strongly symbolic-numeric thinking style with minimal use of visual or spatial representation. S1 tended to depend on memory and rapid arithmetic procedures, which, in this research context, illustrates an efficient yet unidimensional thinking profile. This observation is critical in highlighting the diversity of students' cognitive strategies in understanding and solving trigonometric problems involving beyond-90-degree angles.



The translated problem:
Do you think 570° *is a special angle? Explain your reasoning!*

Figure 3. *Responses of Subject S6*

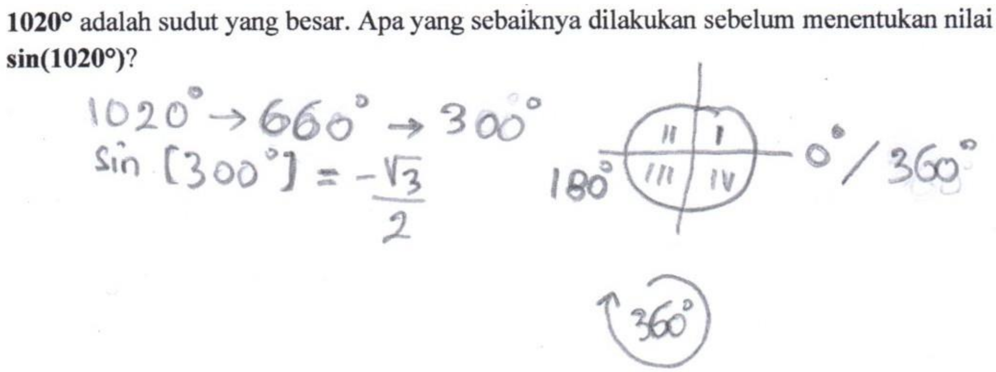
In contrast, as shown in Figure 3, Subject S6 displayed maturity in integrating both visual and symbolic modes of thinking. In response to $\sin(570°)$, S6 quickly recognized that the angle exceeded a full rotation and reduced it to 570° – 360° = 210°. This step was followed by analyzing 210° as 180° + 30°, which indicates an understanding of angle positioning in the third quadrant and the use of reference angles. S6 correctly concluded that $\sin(210°) = -\sin(30°) = -\frac{1}{2}$. During the process, S6 sketched an x-axis and explicitly stated that 210° lies in the third quadrant, where sine values are negative. This demonstrated spatial visualization and quadrant logic within the thinking process. The following think-aloud quote, *"I used the formula of 180 plus a reference angle, because 210 is 180 + 30. Since it's in quadrant III, sine is negative, so I get* $-sin(30°)$*, which is* $-\frac{1}{2}$*,"* affirmed S6's ability to synthesize angle structure knowledge, quadrant logic, and trigonometric values into a systematic reasoning framework. The strategy used reflects advanced visual-mathematical thinking capabilities, aligning with the study's focus on representation and cognitive reduction in solving beyond-90-degree angles trigonometric problems. In the context of this research, S6 stood out as a subject whose spatial and symbolic representations were mutually reinforcing, demonstrating integrative abilities between symbolic procedures, visualization, and fundamental trigonometric concepts.

Table 6. *Summary of DAPIC Analysis for Subjects S1 and S6*

Aspect	Subject S1	Subject S6
Define	Recognized the need to reduce 1080°	Realized 570° exceeded one full rotation and needed to be reduced
Assess	Evaluated that reducing three times 360° was effective	Reduced 360° to get 210°; identified the angle as lying in quadrant III
Plan	Performed direct reduction without visualization	Used the form 180° + 30° to identify quadrant and reference angle
Implement	Executed rapid and accurate computation	Used sketching and symbolic computation: $\sin(210°) = -\sin(30°) = -\frac{1}{2}$
Communicate	Provided a concise but accurate symbolic answer	Explained the negative sign and the value of $\sin(30°)$ using full reasoning
Type of Error	None	No errors; correct process and result
Dominant DAPIC Stage	Complete: D → A → P → I → C	Complete: D → P → I → A → C

Reviewing Table 6 and continuing with point 6, both Subject S1 and Subject S6 demonstrated a strong conceptual grasp of periodicity and quadrant structures in trigonometry. However, their respective approaches and cognitive styles reflected distinct but complementary characteristics within mathematical representation. Subject S1 relied efficiently on symbolic representation, employing rapid mental processing to reduce the large angle (1080°) to an equivalent without explicit visualization (Dehaene et al., 2022). This symbolic dominance indicates strong abstract reasoning and conceptual understanding of trigonometric function periodicity (Tallman, 2021). The systematic and error-free strategy also points to high cognitive efficiency (Pulferer et al., 2024). Conversely, Subject S6 exhibited advanced visual-mathematical thinking. He not only identified the angle structure through the 180° + 30° form but also incorporated spatial sketching and symbolic reasoning that was fully communicated. This approach reflects a more mature integration of spatial and symbolic representation, supporting accurate and logical selection of trigonometric values (Resnick & Lowrie, 2023). Both subjects illustrate diverse but adaptive mathematical thinking profiles suited to beyond-90-degree angles problems. S1 leans towards abstract and symbolic processing, whereas S6 balances visualization and symbolization. These approaches align closely with the study's focus on the significance of representation and visual strategy in simplifying angles and accurately determining trigonometric function values (Liu et al., 2023). Thus, both S1 and S6 make valuable contributions to reinforcing the argument that mastery of representation, whether symbolic or visual, is a key indicator of visual-mathematical thinking (Giyanti & Oktaviyanthi, 2024; Nardi, 2014), particularly in the context of trigonometric problems involving angles greater than 360°.

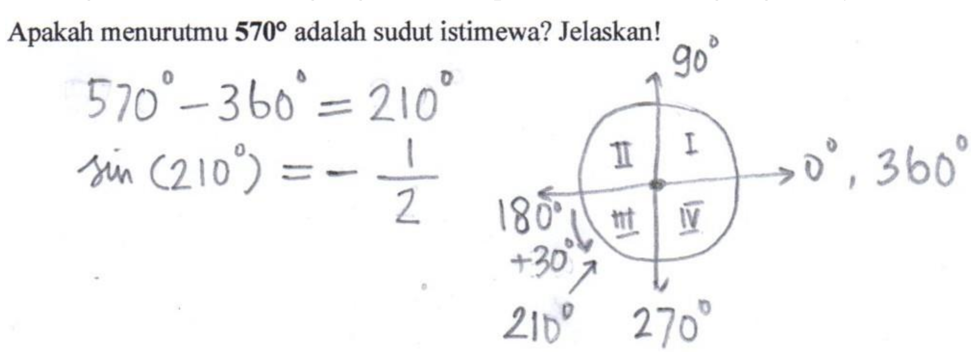
Subjects S3 and S5 – Intermediate Cognitive Level



The translated problem:
$1020°$ *is a large angle. What should be done first before determining the value of* $sin(1020°)$*?*

Figure 4. *Responses of Subject S3*

In Figure 4, Subject S3 demonstrates a systematic and reflective thinking strategy in solving the problem $\sin(1020°)$. They began by gradually reducing the angle, subtracting 360° twice to obtain the equivalent angle 1020° → 660° → 300°. S3 then identified that the angle 300° lies in Quadrant IV and correctly stated that the sine value in that quadrant is negative. The final answer written was $\sin(300°) = -\frac{\sqrt{3}}{2}$. Although S3 showed some hesitation during the reduction from 660° to 300°, they managed to self-correct and exhibited metacognitive awareness during the thinking process. S3 effectively combined periodicity logic, quadrant structure, and knowledge of trigonometric function values. A quote from the think-aloud session, *"1020 is so big… I'll try subtracting 360 twice… I get 300. Oh yeah, 300 is in the fourth quadrant, so sine is negative, so it's minus root three over two,"* illustrates that S3 possesses representational understanding of the unit circle and is capable of navigating large angles in a conceptually sound manner. Although no explicit visual aids were used in the written response, the thinking process indicated strong internal spatial involvement. Overall, S3 demonstrated a balance between procedural and conceptual strategies, with a tendency to verify steps reflectively. This suggests relevant analytical geometric thinking abilities in solving trigonometric problems involving angles beyond one full rotation.



The translated problem:
Do you think $570°$ *is a special angle? Explain your reasoning!*

Figure 5. *Responses of Subject S5*

Meanwhile, Subject S5 in Figure 5 showed an initial understanding of angle reduction and quadrant logic, but experienced uncertainty in precisely determining the sine function value. In solving $\sin(570°)$, they correctly subtracted 360° once to get 570° – 360° = 210°. S5 then stated that $\sin(210°) = -\frac{1}{2}$, reasoning that 210° lies in Quadrant III, where the sine function is negative. Although the value chosen, –½, is indeed correct for $\sin(210°)$, there were indications of uncertainty regarding the reference angle. Their statement reflected doubt, *"I'm not really sure of the exact value, but since 210 is more than 180 and in the third quadrant, the sine should be negative. If the reference angle is 30°, then I think the value is a half."* During the process observation, S5 used their fingers and sketched a unit circle to help determine the angle's position. This shows reliance on visual and kinesthetic representations, although their recall of basic trigonometric values is not yet solid. Subject S5 displayed spatial and conceptual thinking but has not yet fully mastered the connection between reference angles, quadrants, and the exact values of trigonometric functions. The strategy employed is more heuristic in nature rather than mature procedural or symbolic reasoning. Therefore, S5 can be categorized as a subject with emerging visual-conceptual understanding, but still in need of reinforcement in symbolic precision and confidence in recalling trigonometric values.

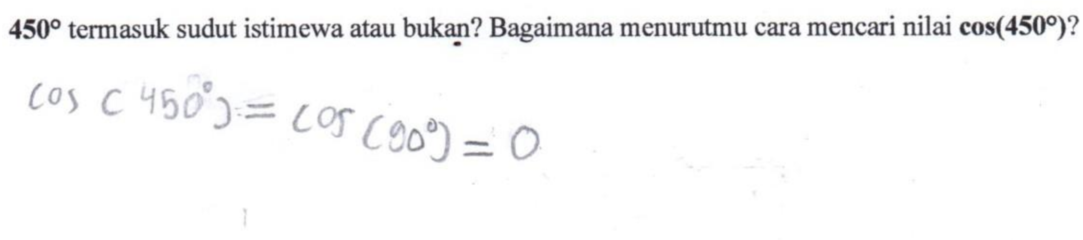
Table 7. *Summary of DAPIC Analysis for Subjects S3 and S5*

Aspect	Subject S3	Subject S5
Define	Recognized and began reducing 1020°	Identified 570° as a large angle that needs reduction
Assess	Evaluated quadrant and angle value	Subtracted 360° → 210°, recognized the angle lies in Quadrant III
Plan	Planned reduction and quadrant validation	Determined the reference angle (30°) and concluded sine is negative in QIII
Implement	Performed reduction and explicit calculation	Drew a unit circle, wrote $\sin(210°) = -\frac{1}{2}$, but was uncertain about the value
Communicate	Explained angle position and sign of function	Explained that sine is negative in Quadrant III, though unsure of function value
Type of Error	None (self-corrected)	Error in selecting $\sin(30°)$ value, not in quadrant or sign identification
Dominant DAPIC Phase	Complete: D → A → P → I → C	Complete: D → A → P → I → C

The DAPIC analysis in Table 7 reveals important contrasts between Subjects S3 and S5 in terms of cognitive performance and representational thinking. Subject S3 displayed inconsistencies in conceptual understanding, particularly in reducing angles and identifying quadrants, leading to a flawed solution path.

The absence of visual representation further hindered the clarity of thought and accuracy (Isaev & Podvesovskii, 2022). S3's errors reflect a deeper issue of not internalizing the periodic structure of trigonometric functions, which is critical for handling angles beyond 360°. Subject S5, on the other hand, showed a more stable command of symbolic manipulation but lacked verification through visual or logical support. Although S5 arrived at the correct answer, the absence of a visual strategy, such as quadrant sketching or reference angle analysis, suggests that the subject relied on rote memorization rather than deep conceptual reasoning. This represents a symbolic-numeric thinking profile with moderate effectiveness and minimal integration with visual strategies (Nardi, 2014; Tallman, 2021). Together, these findings indicate that both S3 and S5 occupy an intermediate cognitive level, with partially formed trigonometric reasoning and limited use of multiple representations. Their strategies show potential but fall short of achieving the integrative mathematical thinking demonstrated by subjects at higher cognitive levels. These observations affirm the significance of encouraging visual strategies and quadrant-based reasoning in trigonometric problem-solving involving beyond-90-degree angles.

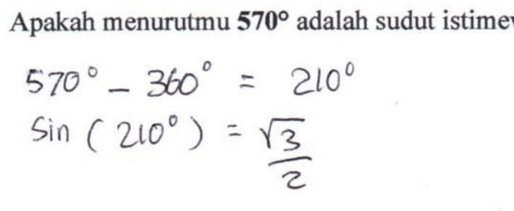
Subjects S2 and S4 – Low Cognitive Performance Level



The translated problem:
Is 450° considered a special angle or not? In your opinion, how can we find the value of $\cos(450°)$*?*

Figure 6. *Responses of Subject S2*

Subject S2 in Figure 6 demonstrated a thinking process dominated by spatial intuition without the support of explicit symbolic or numerical representations. When solving the problem of $\cos(450°)$, S2 immediately stated that the result was the same as $\cos(90°)$ and gave the value as 0. Although the final answer was correct, there was no written or explained process of angle reduction from a mathematical standpoint. This indicates that S2 did not actively use numerical strategies such as subtracting multiples of 360°, which are typically foundational in solving beyond-90-degree angles trigonometric problems. Observations during the problem-solving process showed that S2 appeared hesitant and confused by the unusual angle. They relied on visual reasoning, saying *"450 degrees is like turning back upwards,"* suggesting an intuitive understanding of the circular shape of the unit circle, but an inability to explain quadrant transitions or angle positions mathematically. The think-aloud quote, *"I think it's the same as 90 degrees... like turning back up again,"* reinforces the conclusion that S2's spatial representation is not yet well-integrated with symbolic or procedural systems. This is a critical point in the context of the study, indicating that S2’s representational strategy is imaginative but insufficiently validated, highlighting the need to strengthen the ability to connect spatial intuition with formal procedures in trigonometry learning.



The translated problem:
Do you think 570° *is a special angle? Explain your reasoning!*

***Figure** 7. Responses of Subject S4*

In contrast to S2, Subject S4 (Figure 7) appeared to employ a procedural approach when working on the problem $\sin(570°)$, but made an error during the interpretation stage. S4 subtracted 360° from 570° to obtain 210°, an initial step that shows an understanding of the need to reduce large angles. However, when determining the sine value, S4 stated that $\sin(210°) = -\frac{\sqrt{3}}{2}$ and incorrectly believed the value to be positive. This indicates a fundamental misunderstanding of trigonometric function signs based on quadrants. In fact, 210° lies in the third quadrant, where the sine function should be negative, and the correct sine value of a 30° reference angle is $\frac{1}{2}$, not $-\frac{\sqrt{3}}{2}$. During observations, S4 did not appear to explicitly consider the quadrant system, and statements such as *“sine is usually positive”* suggest that the reasoning used was more generalized and not concept-based. Although they were able to identify the reference angle as 30°, they failed to connect the angle’s position with the rules governing trigonometric function signs. This reflects a gap between numerical understanding and spatial-conceptual understanding. Thus, S4 exhibited a different type of error compared to S2, focusing more on interpretation and conclusion-making, which is also an important aspect highlighted in the study concerning answer validation, quadrant use, and visual representation as part of beyond-90-degree angles trigonometry problem-solving strategies.

From an instructional perspective, S2's reliance on spatial intuition without symbolic justification suggests the need for scaffolded tasks that connect intuitive visuals with formal procedures—such as guided unit circle labeling exercises. Meanwhile, S4’s sign error in the third quadrant calls for targeted reinforcement of sign conventions through quadrant-based sorting activities and metacognitive prompts focused on angle location analysis.

Table 8. Summary of DAPIC Analysis for Subjects S2 and S4

Aspect	Subject S2	Subject S4
Define	Recognizes that 450° > 360°, but without analysis	Recognizes that 570° needs to be reduced
Assess	No explicit evaluation performed	Reduction done, but function sign not evaluated
Plan	No explicit strategy developed	No quadrant validation plan formulated
Implement	Guesses directly without procedure	Correct arithmetic reduction, incorrect sign
Communicate	Very limited explanation	General argument, not concept-based
Error Type	Hidden conceptual (reduction not explicit)	Incorrect sine sign (third quadrant)
Dominant DAPIC Stages	Limited: D → P → C	Incomplete: D → A → P → I

Based on Table 8, both subjects, S2 and S4, showed different dynamics in their thinking processes when solving trigonometry problems involving beyond-90-degree angles, yet both experienced conceptual obstacles that reflect common challenges in learning this topic. Subject S2 showed that although they could produce the correct final answer, the strategy used was very limited and not based on complete mathematical procedures or understanding. They did not write down the process of reducing the 450° angle, did not evaluate the angle's position in the unit circle, and did not use other visual or numerical representation schemes. This suggests that S2's understanding remains intuitive and non-reflective, addressing the study's focus on the importance of representational validation in solving trigonometric problems (Ramírez-Uclés & Ruiz-Hidalgo, 2022). The limited DAPIC stages observed in S2, only Define, Plan, and minimally Communicate, indicate that students like S2 tend to arrive at answers based on spatial guessing alone, without explicit support from symbolic systems or quadrant understanding. Meanwhile, S4 showed a better understanding of the initial procedure, such as reducing the angle (570° to 210°), but made a mistake in determining the sine value due to overlooking the sign system of functions based on quadrants. Although they understood that 210° references 30°, they incorrectly stated the sine value as positive and wrote $-\frac{\sqrt{3}}{2}$ as the answer, whereas the correct value is $\frac{1}{2}$. This shows that although procedural steps were attempted, S4 was unable to validate the result, particularly in terms of signs and reference angles in trigonometric functions. Such errors are highly relevant to the study's focus, which highlights the types of mistakes students make when solving beyond-90-degree angles problems, especially errors in using the quadrant system (Tallman, 2021). S4 exhibited a more complete DAPIC trail than S2, but it remained incomplete due to a lack of validation (I) and concept-based communication (C). These two cases comparatively highlight the importance of strengthening the connection between symbolic representation, spatial visualization, and mathematical validation strategies, which form the core of this investigation (Dehaene et al., 2022; Resnick & Lowrie, 2023).

Among the six subjects analyzed, it was found that the ability to properly reduce angles, understand trigonometric function signs across different quadrants, and use diverse representations (verbal, visual, symbolic) were key factors in successfully solving beyond-90-degree angles problems. High-cognitive-level subjects (S1, S6) consistently demonstrated answer validation, conceptual understanding, and strong metacognitive control. Meanwhile, low-cognitive-level subjects (S2, S4) tended to make recurring errors without improvement strategies. These findings address the main research question concerning how variations in students' thinking processes, errors, and representation strategies emerge when solving trigonometric functions involving angles greater than 90°, and how cognitive levels are related to the quality of problem-solving. To facilitate a clearer alignment between the findings and the three research objectives, Table 9 provides a thematic synthesis of students' thinking patterns, types of representations employed, and recurring conceptual difficulties across each DAPIC stage. This integration offers a cross-subject view that complements the case-based narratives and supports readers in synthesizing key insights more efficiently.

Table 9. Thematic Synthesis of Thinking Patterns

DAPIC Stage	Thinking Patterns	Type of Representations Used	Common Errors/ Strategies	Subjects
Define	Recognizing need to reduce angle; initial classification of angle magnitude	Primarily symbolic (e.g., subtraction of 360°); some verbal reasoning	Misidentification of angle category (e.g., special vs. general angle); skipping definition step entirely	S2 (skipped), S4 (partial), S5 (correct but hesitant)
Assess	Analyzing quadrant location and reference angle; validating angle positioning	Spatial (e.g., sketching unit circle); verbal explanation; limited graphical representation	Failure to determine quadrant (S4); confusion between quadrant and function sign; incomplete evaluation	S3 (minor hesitation), S4 (error), S5 (uncertain)
Plan	Strategy selection for reduction and reference angle transformation	Symbolic plans (e.g., 180° + 30°), sketches, mental visualization	Absence of explicit strategy (S2); inconsistent strategy-to-quadrant matching	S2 (none), S5 (partial), S6 (integrative)
Implement	Executing computation with symbolic or visual support	Symbolic computation, finger tracing, unit circle sketches	Arithmetic performed without validation; over-reliance on intuition without structure	S2 (guessing), S4 (sign error), S6 (accurate)
Communicate	Justifying answer; explaining reasoning process; error-checking	Verbal explanations; symbolic summary; minimal graphical elaboration	Justification missing or superficial; partial or incorrect verbal logic	S1 (concise), S2 (limited), S4 (incorrect logic)

As shown in Table 9, different stages of DAPIC reveal specific cognitive tendencies and challenges. For example, students like S2 and S4 struggled with conceptual validation during the Assess and

Communicate stages, often due to inadequate quadrant reasoning or incomplete explanation. Meanwhile, high-performing students such as S1 and S6 displayed strategic planning and symbolic-visual integration, leading to correct solutions with minimal error. These patterns confirm the interconnectedness between representational fluency and the quality of problem-solving strategies.

The analysis of six participants (S1–S6) revealed diverse thinking patterns as they engaged with trigonometric problems involving extended angles beyond 360°. These patterns such as defined as students' recurring problem-solving strategies, justification styles, and transitions between representations, were interpreted through the DAPIC framework (Define, Assess, Plan, Implement, Communicate). Triangulated data sources, including students' written responses, think-aloud protocols, semi-structured interviews, observation sheets, and audio-video recordings, enabled a contextualized and multifaceted understanding of their cognitive processes. The data showed how students' approaches diverged based on cognitive levels, representational preferences, and their engagement with specific DAPIC stages. To enhance accessibility and synthesis, Table 9 presents a thematic summary of the thinking patterns, representations used, and common conceptual errors across DAPIC stages and student profiles. The following discussion elaborates on these themes by integrating key findings per DAPIC stage, thereby aligning the narrative with the three research objectives: identifying thinking patterns, exploring representational strategies, and diagnosing conceptual difficulties.

In the **Define** phase, high-performing students like S1 and S6 were able to recognize the need to reduce angles beyond 360° into reference angles within a standard interval, using symbolic strategies such as subtracting multiples of 360°, alongside verbal articulation of their reasoning. Their ability to fluently recognize and restructure extended-angle problems indicates robust conceptual understanding. In contrast, S2 skipped the definition phase altogether, directly attempting procedural steps without first reducing or classifying the angle. Interview data confirmed that S2 was unsure whether a 450° angle required simplification, reflecting a fundamental misunderstanding of angle periodicity. This finding affirms the importance of structured recognition processes in trigonometric problem-solving, as emphasized in prior research (Tallman, 2021), and highlights how failure at the initial phase can cascade into subsequent errors.

The **Assess** phase required students to identify the appropriate quadrant and understand the sign of trigonometric functions within that quadrant. Observation sheets and video data revealed that high performers (e.g., S6) used sketches of the unit circle or gestural reasoning to correctly determine quadrant placement. Meanwhile, students like S4 and S2 displayed confusion between the reference angle and quadrant, with S4 assigning the wrong sign to a trigonometric value due to incorrect quadrant identification. Think-aloud data from S4 showed verbal conflict between their symbolic reasoning and visual understanding, indicating cognitive dissonance. These findings support the theory of multiple representations (S. Ainsworth, 1999; S. E. Ainsworth & Scheiter, 2021), which emphasizes the importance of integrating visual and symbolic resources to achieve accurate spatial reasoning. The difficulty in quadrant determination also aligns with (Santos-Trigo, 2020), who found that spatial misjudgement is a major barrier in trigonometric learning.

During the **Plan** phase, the strategic variability among students became apparent. Students such as S1 and S3 verbalized and wrote out a clear plan, often integrating symbolic expressions like "180° + 30°" with spatial intuition to guide their next steps. In contrast, S2 and S5 showed either a lack of explicit planning or inconsistencies in matching reference angles with correct quadrants. The observation sheets documented moments where S5 hesitated before drawing the unit circle but proceeded without confirming accuracy. According to (Sweller, 2022) cognitive load theory, this suggests that limited working memory may have hindered their ability to coordinate strategy selection and quadrant identification, particularly when procedural knowledge was not fully automated.

The **Implement** stage focused on the execution of calculations and graphical solutions. High-achieving students relied on symbolic computation alongside visual representations. For example, S6 used finger tracing on the drawn unit circle to verify symbolic results, demonstrating dual-channel verification. Meanwhile, S4 and S2 performed computations without validating the sign or quadrant, relying purely on memorized procedures. Video recordings confirmed a lack of correction behaviours in these students—gestures were abrupt and did not correspond to reflective checks. These results corroborate (Wallsten, 1980) information processing theory, which emphasizes the role of monitoring and control in successful problem-solving.

In the **Communicate** phase, clear differences in metacognitive expression were evident. Students like S1 and S6 were able to explain their reasoning verbally and justify their answers with reference to their earlier decisions. Audio-video recordings showed that they paused before explaining, indicating reflective monitoring. By contrast, students like S2 and S4 either gave no explanation or repeated procedural steps without connecting them to conceptual reasoning. This pattern is consistent with Metacognitive Theory (Schraw & Moshman, 1995), which stresses that articulation and justification are indicators of deep understanding. The results also resonate with findings by (Meier et al., 1996) and von (von Thienen et al., 2023), who note that metacognitive engagement is essential in tasks requiring high representational coordination.

Instructionally, the findings suggest targeted scaffolds are needed for students like S2 and S4. For instance, S2's failure in the Define and Plan stages could be addressed by incorporating digital learning tools that emphasize angle reduction through animated transformations or guided prompts. S4's errors in quadrant and sign assignment indicate a need for spatial reinforcement, such as using color-coded unit circles or quadrant-labeling exercises. This aligns with recommendations by (Nielsen & Bostic, 2018) and supports the integration of dynamic visual tools like GeoGebra and PhET simulations (Haleva et al., 2021; Sholahudin & Oktaviyanthi, 2025), which can help bridge the gap between symbolic and spatial understanding.

Finally, students' performance across DAPIC stages reflected their broader cognitive profiles. High-performing students (S1, S3, S5, S6) demonstrated fluent representational shifts and strategic flexibility, while low-performing students (S2, S4) showed fragmented or rigid strategies. The ability to self-regulate errors, explain reasoning, and apply consistent strategies was concentrated among students with higher cognitive scores on the diagnostic pretest. These results reinforce the alignment between DAPIC implementation and theories of cognitive development, including cognitive load theory (Sweller, 2011, 2020, 2022) and information processing models (Wallsten, 1980). Moreover, the process-oriented application of DAPIC in this study presents a methodological contribution by offering a structured lens to identify specific cognitive breakdowns and plan targeted interventions in trigonometric instruction.

Conclusion

This study investigated students' cognitive processes in solving trigonometric problems involving angles beyond 90°, using the DAPIC framework (Define, Assess, Plan, Implement, Communicate). By mapping students' thinking patterns across each DAPIC stage and triangulating data through written responses, think-aloud protocols, interviews, and video observations, the study uncovered key variations in how students approach extended-angle problems depending on their cognitive level. The key contribution of this research lies in its novel application of the DAPIC framework to the domain of extended-angle trigonometry, a topic that has rarely been analyzed through such structured cognitive lenses. By reconstructing students' real-time problem-solving trajectories, the study provides insight into how conceptual errors and representational choices emerge across different stages of reasoning. The findings reinforce and extend existing theoretical frameworks, including Duval's Multiple Representation Theory, Cognitive Load Theory, and metacognitive models, by showing how representational fluency and cognitive control co-develop during complex mathematical tasks. These insights highlight the importance of guiding students through structured problem-solving sequences. High-performing students consistently engaged in all five DAPIC stages, integrating symbolic, visual, and verbal forms of reasoning that supported accuracy and reflective justification. In contrast, students with lower levels of cognitive engagement tended to bypass the Define or Assess stages, or relied solely on symbolic manipulation, which led to fragmented reasoning and common conceptual errors, particularly in quadrant identification, angle reduction, and the interpretation of undefined values.

From a practical standpoint, the DAPIC framework can serve as a pedagogical scaffold for designing both classroom instruction and assessment. Teachers can embed each DAPIC stage into their lesson design, for instance by prompting angle classification and prediction in the Define stage, using unit circle visuals for quadrant analysis in the Assess stage, encouraging explicit planning before execution, and incorporating peer justification during the Communicate phase. Incorporating multimodal learning tools, such as GeoGebra simulations, interactive digital worksheets, and color-coded visual aids, can further support students' development of representational fluency. Diagnostic pre-assessments may also help identify cognitive readiness and allow educators to tailor interventions accordingly. Looking ahead, future research may explore the integration of neuro-adaptive technologies such as EEG-based monitoring to track students' cognitive load in real time, particularly during challenging tasks involving extended trigonometric reasoning. The DAPIC framework also holds potential as a formative assessment model, allowing educators to diagnose and respond to students' cognitive profiles with targeted instructional strategies that evolve in response to real-time learning data.

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